MATHEMATICS

FORM ONE

NOTES

NUMBERS

We know that when we count we start 1,2 .... . But there are other numbers like 0, negative numbers and decimals. All these types of numbers are categorized in different groups like counting numbers, integers,real numbers, whole numbers and rational and irrational numbers according to their properties. all this have been covered in this chapter

Base Ten Numeration

Numbers are represented by symbols called numerals. For example, numeral for the number ten is 10. Numeral for the number hundred is 110 and so on.

The symbols which represent numbers are called digits. For example the number 521 has three (3) digits which are 5, 2 and 1. There are only tendigits which are used to represent any number. These digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

The Place Value in each Digit in Base Ten Numeration

Identify the place value in each digit in base ten numeration

When we write a number, for example 521, each digit has a different value called place value. The 1 on the right means 1 ones which can be written as 1×1, the next number which is 2 means 2 tens which can be written as 2×10 and the last number which is 5 means 5 hundreds which can be written as 5×100. Therefore the number 521 was found by adding the numbers $5 \times 100 + 2 \times 10 + 1\times 1 = 521$.

Note that when writing numbers in words, if there is zero between numbers we use word ‘and’

Example 1

Write the following numbers in words:

1. 7 008

2. 99 827 213

3. 59 000

Solution

a. 7 008 = Seven thousand and eight.

b. 99 827 213 = Ninety nine millions eight hundred twenty seven thousand two hundred thirteen.

c. 59 000 = Fifty nine thousand.

Example 2

Write the numbers bellow in expanded form.

1. 732.

2. 1 205.

Solution

a. 732 = 7 x 100 + 3 x 10 + 2 x 1

b. 1 205 = 1 x 1000 + 2 x 100 + 0 x 10 + 5 x 1

Example 3

Write in numerals for each of the following:

1. 9 x 100 + 8 x 10 + 0 x 1

2. Nine hundred fifty five thousand and five.

Solution

a. 9 x 100 + 8 x 10 + 0 x 1 = 980

b. Nine hundred fifty five thousand and five = 955 005.

Example 4

For each of the following numbers write the place value of the digit in brackets.

1. 89 705 361 (8)

2. 57 341 (7)

Solution

a. 8 is in the place value of ten millions.

Billions	Hundred millions	Ten millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
									1
								1	0
							1	0	0
						1	0	0	0
					1	0	0	0	0
				1	0	0	0	0	0
			1	0	0	0	0	0	0
		1	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0

b. 7 is in the place value of thousands.

Numbers in Base Ten Numeration

Read numbers in base ten numeration

Base Ten Numeration is a system of writing numbers using ten symbols i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Base Ten Numeration is also called decimal system of Numeration.

Numbers in Base Ten Numeration up to One Billion

Write numbers in base ten numeration up to one billion

Consider the table below showing place values of numbers up to one Billion.

If you are given numerals for a number having more than three digits, you have to write it by grouping the digits into groups of three digits from right. For example 7892939 is written as 7 892 939.

When we are writing numbers in words we consider their place values. For example; if we are told to write 725 in words, we first need to know the place value of each digit. Starting from right side 5 is in the place value of ones, 2 is in the place value of tens and seven is in the place value of hundreds. Therefore our numeral will be read as seven hundred twenty five.

Numbers in Daily Life

Apply numbers in daily life

Numbers play an important role in our lives. Almost all the things we do involve numbers and Mathematics. Whether we like it or not, our life revolves in numbers since the day we were born. There are numerous numbers directly or indirectly connected to our lives.

The following are some uses of numbers in our daily life:

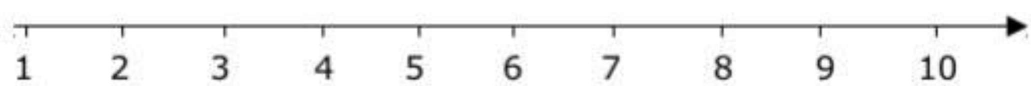
1. Calling a member of a family or a friend using mobile phone.
2. Calculating your daily budget for your food, transportation, and other expenses.
3. Cooking, or anything that involves the idea of proportion and percentage.
4. Weighing fruits, vegetables, meat, chicken, and others in market.
5. Using elevators to go places or floors in the building.
6. Looking at the price of discounted items in a shopping mall.
7. Looking for the number of people who liked your post on Facebook.
8. Switching the channels of your favorite TV shows.
9. Telling time you spent on work or school.

10. Computing the interest you gained on your business.

Natural and Whole Number

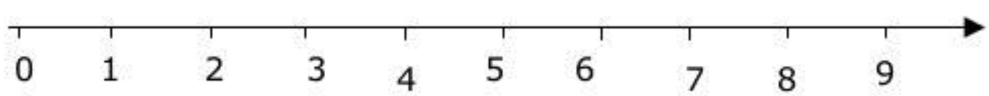
Natural numbers are counting numbers. Counting always starts with 1 and continues with 2, 3 … (dots means numbers continue with no end). Therefore Natural or counting numbers starts from 1 and continue to infinite (no end).

Natural numbers are denoted by **N**. natural numbers can be represented on a horizontal line called number line as shown above:



The other group of numbers starts from zero and it is called whole numbers denoted by **W**.

We can represent whole numbers on a number line as shown above:



The Difference between Natural and Whole Numbers

Distinguish between natural and whole numbers

Natural numbers are either used to count one to one objects or represent the position of an object in a sequence. They start from one and go on to infinity.This is why they are sometimes referred to as counting numbers. The only whole number that cannot be classified as a natural number is 0. Counting numbers can further be classified into perfect numbers, composite numbers, co-prime/ relatively prime numbers, prime numbers, even and odd numbers.

Even ,Odd, and Prime Numbers

Identify even ,odd, and prime numbers

Even numbers are those numbers which are divisible by 2. In other words we can say that any natural number which can be divided by 2 is an even number. For example 2, 4, 6, 8, 10, 12, … are even numbers since they are divisible by 2.

Odd numbers are those numbers which are not divisible by 2. In other words we can say that any natural number when divided by 2 and remains 1 including 1 are called odd numbers. Example 1, 3, 5, 7, 9, 11, 13, 15, … are called odd numbers.

Prime numbers are those numbers which are divisible by one and itself excluding one or any natural number which is divisible by one and itself except one. For example 2, 3, 5, 7, 11, 13, 17, … are called prime numbers.

Odd and Prime Numbers on Numbers Lines

Show even , odd and prime numbers on number lines

Odd and Prime Numbers

Prime and Odd Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Operations with Whole Numbers

We have four operations which are: addition (+), subtraction (-), multiplication (X) and division (÷).

Addition of Whole Numbers

Add whole numbers

When adding numbers we add the corresponding digits in their corresponding place values and we start adding from the right side i.e. from the place value of ones to the next.

We can add numbers horizontally or vertically.

Horizontal addition

Example 5

1. 972 + 18=
2. 23 750 + 250 =

Solution

1. 972 + 18 = 990
2. 23 750 + 250 = 24 000

Vertical addition

Example 6

i. 2 378 943
+ 72 812

______

ii. 7 250
+ 2 352 890

______

Solution

i. 2 378 943
+ 72 812
2 451 755

ii. 7 250
+ 2 352 890
2 360 140

Subtraction of Whole Numbers

Subtract whole numbers

Subtraction is denoted by the sign (-). It is sometimes called **minus**. Subtraction is the opposite of addition. Subtraction also means reduce a number from certain number and the answer that is obtained is called **difference.**.

Subtraction is done in similar way like addition. We subtract the corresponding digits in their corresponding place value. For example; 505 – 13. We first subtract ones, which are 5 and 3. Subtract3 from 5 gives 2. Followed by tens which are 0 and 1. Subtract 1 from 0 is not possible. In order to make it easy, take 1 from 5 (hundreds). When 1 is added to 0 it has to be changed to be tens since it is added to a place of tens. So, when 1 comes into a place of tens it becomes 10. So add 10 to 0. We get 10. Now, subtract 1 from 10. We get 9. We are left with 4 in a place of hundreds since we took 1. There for our answer will be 492.

Note that similar manner will be used when subtracting.

Example 7

a. 89 237 – 2 275 =

b. 8 927 209

   - 997 100

   ____

Solution

a. 89 237 – 2 275 = 86 962

b. 8 927 209

   - 997 100

   930 109

Multiplication of Whole Numbers

Multiply whole numbers

Multiplication means adding repeatedly depending on the times number given. For example; 25 6 means add 25, repeat adding 6 times i.e. 25 + 25 + 25 + 25 +25 + 25 = 150. The answer obtained after multiplying two or more numbers is called **product**. The number being multiplied is called a **multiplicand** while the number used in multiplying is called a **multiplier**. Referring our example, 25 is multiplicand and 6 is multiplier.

Example 8

1. $925 \times 35 =$
2. $$\begin{array}{r} 752\,345 \\ \times \quad 15 \\ \hline \\ \hline \end{array}$$

Solution

1. $925 \times 35 = 32\,375$

2. $$\begin{array}{r} 752\,345 \\ \times \quad 15 \\ \hline 752345 \\ +3761725 \\ \hline 11285175 \\ \hline \end{array}$$

Division of Whole Numbers

Divide whole numbers

Division is the same as subtraction. You subtract**divisor**(the number used to divide another number) from**dividend**(the number which is to be divided), we repeat subtracting divisor to the answer obtained until we get zero. The answer is how many times you repeat subtraction.

For example; 27 ÷9, we take 27 we subtract 9, we get 18. Again we take 18 we subtract 9, we get 9. We take 9 we subtract 9 we get 0. We repeat subtraction three times. Therefore the answer is 3. The answer obtained is called**quotient**. Referring to our example; 27 is dividend, 9 is divisor and 3 is quotient. If a number can't be divided exactly, what remains or left over is called**remainder**.

Example 9

1. 1 714 608 ÷ 18 =

2. 12)12 750

Solution

1) 1 714 608 × 18 = 95 256

```
     1062 rem 6
2) 12)12 750
      -12
        75
       -72
         24
          6
```

The Four Operations in Solving Word Problems

Use the four operations in solving word problems

Sometimes you may be given a question with mixed operations +, -, xand ÷ . We do multiplication and division first then addition and subtraction.

Example 10

1. 12 ÷ 4 + 3 x 5

2. 14 x2 ÷ 7 – 3 + 6

Solution

1. 12 x 4 + 3 x 5 =3 + 15 (do division and multiplication fist) =18

2. 14 x 2 ÷ 7 – 3 + 6 =28 ÷ 7 - 3 + 6 (multiply first) =4 – 3 + 6 (then divide) =10 – 3 (add then subtract) =7

We may use brackets to separate x,÷ , + and – if they are mixed in the same problem and use what is called BODMAS . BODMAS is the short form of the following:

B for Brackets O for Open D for Division M for Multiplication A for Addition and S for Subtraction

Therefore, with mixed operations, we first do the operation inside the brackets; we say that we open the brackets. Then we do division followed by multiplication, addition and lastly subtraction.

Example 11

1. $(78 \div 3 + 4) \div (2 \times 3) - 2$
2. $4 + 3 - (5 - 3) + 8 \div (9 - 7)$

Solution

1. $(78 \div 3 + 4) \div (2 \times 3) - 2$

$=30 \div 6 - 2$ (first do the operations inside the brackets)

$=5 -2$ (do division)

$=3$

2. $4 + 3 - (5 - 3) + 8 \div (9 - 7)$

$=4 + 3 - 2 + 8 \div 2$(do the operations inside the brackets first)

$=4 + 3 -2 + 4$ (do division)

$=11 - 2$ (do addition and then subtraction)

$=9$

Word problems on whole numbers

Example 12

In a school library there are 6 shelves each with 30 books. How many books are there?

Solution

Each shelf has 30 books

6 shelves have 30 × 6 = 180 books.

Therefore, there are 180 books.

Example 13

Juma's mother has a garden with Tomatoes, Cabbages and Water Lemons. There are 4 rows of Tomato each with 30 in it. 6 rows of Cabbages with 25 in each and 3 rows of Water Lemo

Solution

There are 30 Tomatoes in each row

4 rows will have 30 × 4 = 120 Tomatoes

Each row has 25 Cabbages

6 rows have 25 × 6 = 150 Cabbages

Each row has 15 Water Lemons

3 rows have 15 × 3 = 45 Water Lemons

In total there 120 + 150 + 45 = 315 plants.

Therefore in Juma's mother garden there are 315 plants.

Example 14

A school shop collects sh 90 000 from customers each day. If sh 380 000 from the collection of 6 days was used to buy books. How much money was left?

Each day the collection is sh 90 000

6 days collection is sh 90 000 × 6 = sh 540 000

The money left will be = Total collection – Money used

= sh 540 000 – sh 380 000 = sh 160 000

Therefore the money left was sh 160 000

Exercise 1

1. For each of the following numbers write the place value of a digit in a bracket.

a. 899 482 (4)

b. 1 940 (0)

c. 9 123 476

2. Write the numerals for each of the following problems.

a. Ten thousand and fifty one.

b. Nine hundred thirty millions one hundred twenty five thousand three hundred seventy four.

c. 6 x 100 + 1 x 10 + 7 x 1

d. 5 x 10 000 + 4 x 1 000 + 2 x 100 + 7 x 10 + 8 x 1

3. Write the following numerals in words

a. 952 817

b. 98 802 750

4. Write down even, odd and prime numbers between 90 and 100.

5. Compute:

a. 25 940 + 72 115 – 5 750 =

b. 892 x 12=

c. 14 670 ÷ 15 =

6. Calculate: (75 ÷3) + 7 – 14 + (13 x2) =

7. There are 23 streams at Mamboleo primary school. If each stream has 60 pupils except 4 streams which they have 75 pupils each. How many pupils are there at Mamboleo primary school?

8. Hamis uses 200 shillings every day for transport. How much money will he use for 35 days?

Factors And Multiples Of Numbers

Factors of a Number

Find factors of a number

Consider two numbers 5 and 6, when we multiply these numbers i.e. 5 6 the answer is 30. The numbers 5 and 6 are called factors or divisors of 30 and number 30 is called a multiple of 5 and 6. Therefore factors are the divisors of a number.

Example 15

Find all factors of 12

Note that, when listing the factors we don't repeat any of it.

Write 12 as a product of two numbers

$$
\begin{aligned}
12 &= 1 \times 12 \\
&= 2 \times 6 \\
&= 3 \times 4 \\
&= 4 \times 3 \\
&= 6 \times 2 \\
&= 12 \times 1
\end{aligned}
$$

Therefore, the factors of 12 are 1, 2, 3, 4, 6, 12.

Note that, when listing the factors we don't repeat any of it.

Multiples of a Number

Find multiples of a number

Multiples of a number are the products of a number. For example multiples of 4 are 4, 8, 12, 16, 20, 24, … This means multiply 4 by 1, 2, 3, 4, 5, 6, … .

Example 16

Example

List all multiples of 6 between 30 and 45.

Solution

The multiples of 6 are: 6×1 = 6; 6×2 = 12; 6×3 = 18; 6×4 = 24; 6×5 = 30; 6×6 = 36; 6×7 = 42; 6×8 = 48 and so on.

Therefore, multiples of 6 between 30 and 45 are 36 and 42.

Factors to Find the Greatest Common Factors(GCF) of Numbers

Use factors to find the greatest common factors(GCF) of numbers

Greatest Common Factor is sometimes called Highest Common Factor. Its short form is (GCF) or (HCF) respectively. The Greatest Common Factor is the largest common divisor of two or more numbers given.

For example if you are told to find the GCF of 15 and 25. First, list all factors or divisors of 15 and that of 25. Thus, factors of 15 are 1, 3, 5, 15factors of 25 are 1, 5, 25

The common factors are 1 and 5. Therefore the GCF is 5.

Example 17

Example1

Find the HCF of 72 and 120.

Solution

We have to list factors of our numbers:

Factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Factors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

The common factors are 1, 2, 3, 4, 6, 8, 12, 24.

Therefore the HCF of 72 and 120 is 24.

Another method which can be used to find the GCF or HCF is prime factorization method.

Example 18

Example

Find the HCF of 36 and 48.

Solution

We have to find prime factors of 36 and 48 first

36	48
2) 36	2)48
2)18	2)24
3)9	2)12
3)3	2) 6
1	3) 3
	1

thus, 36 = 2×2×3×3.

48 = 2×2×2× 2×3.

After writing the numbers as a product of their prime factors, take only the common prime factors (prime factors appeared to all numbers). In our example the common factors are 2×2×3. Therefore the HCF of 36 and 48 is 12.

Lowest Common Multiple (LCM)

Lowest Common Multiple is also called Least Common Multiple and its short form is LCM.For example: multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, …multiples of 3 are 3, 9, 15, 18, 21, 24, 27, 30, 33, ….

When you look carefully at these multiples of 5 and 3 you notes that 15 had appeared to both. This multiple which appear to both is called a common multiple. If there are more than one common multiples which appeared the smaller common multiple is what is called Lowest Common Multiple.

Example 19

Find the common multiples and then show the Lowest Common Multiple of the numbers 4, 6 and 8.

Solution

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, …

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, …

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, …

The common multiples of 4, 6 and 8 are 24, 48, …

Therefore the Lowest Common Multiple of 4, 6, and 8 is 24.

We can find the Lowest Common Multiple (LCM) of numbers by writing the numbers as a product of their prime factors. The method is called prime factorization.

Example 20

Find the LCM of 24 and 36 by prime factorization method.

Solution

Let us find prime factors of each number by dividing the numbers by their prime factors.

```
2) 24        2)36
2)12         2)18
2)  6        3)  9
3)  3        3)  3
     1            1
```

Thus, $24 = 2 \times 2 \times 2 \times 3$.

$36 = 2 \times 2 \times 3 \times 3$.

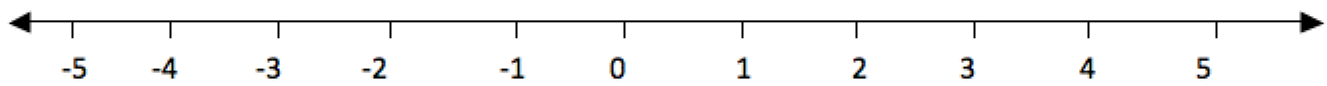
Now take the prime factors which appear to both numbers i.e. 2×2×3 (we take without repeating). We are left with 2 which is a multiple of 24 and 3 which is a multiple 36. We have also to multiply these prime factors left i.e. 2×2×2×3×3. This gives 72. Therefore the LCM of 24 and 36 is 2×2×2×3×3= 72.

Integers

Integers

Identify integers

Consider a number line below



The numbers from 0 to the right are called positive numbers and the numbers from 0 to the left with minus (-) sign are called negative numbers. Therefore all numbers with positive (+) or negative (-) sign are called integers and they are denoted by Z. Numbers with positive sign are written without showing the positive sign. For example +1, +2, +3, … they are written simply as 1, 2, 3, … . But negative numbers must carry negative sign (-). Therefore integers are all positive and negative numbers including zero (0). Zero is neither positive nor negative number. It is neutral.

The numbers from zero to the right increases their values as the increase. While the numbers from zero to the left decrease their values as they increase. Consider a number line below.

If you take the numbers 2 and 3, 3 is to the right of 2, so 3 is greater than 2. We use the symbol ‘>’ to show that the number is greater than i. e. 3 >2(three is greater than two). And since 2 is to the left of 3, we say that 2 is smaller than 3 i.e. 2<3 .="" div="" is="" less="" number="" show="" symbol="" than.="" that="" the="" to="" use="">

Consider numbers to the left of 0. For example if you take -5 and -3. -5 is to the left of -3, therefore -5 is smaller than -3. -3 is to the right of -5, therefore -3 is greater than -5.

Generally, the number which is to the right of the other number is greater than the number which is to the left of it.

If two numbers are not equal to each to each other, we use the symbol '≠' to show that the two numbers are not equal. The not equal to '≠' is the opposite of is equal to '='.

Example 21

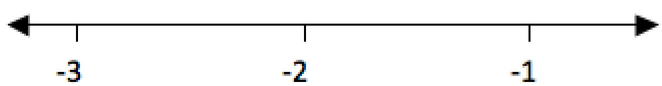
Represent the following integers Z on a number line

a. 0 is greater than Z and Z is greater than -4

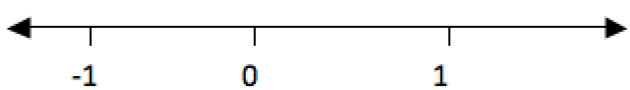
b. -2 is less than Z and Z is less than or equal to 1.

Solution

a. 0 is greater than Z means the integers to the left of zero and Z is greater than -4 means integers to the left of -4. These numbers are -1, -2 and -3. Consider number line below



b. -2 is less than Z means integers to the right of -2 and Z is less than or equal to 1 means integers to the left of 1 including 1. These integers are -1, 0 and 1. Consider the number line below



Example 22

Put the signs 'is greater than' (>), 'is less than' (<), 'is equal to' (=) to make a true statement.

a.	-4	☐	-5	d.	78	☐	78
b.	7	☐	7	e.	-25	☐	-37
c.	-32	☐	-3	f.	20	☐	5

Solution

a. -4 $\boxed{>}$ -5 d. 78 $\boxed{=}$ 78

b. 7 $\boxed{=}$ 7 e. -25 $\boxed{>}$ -37

c. -32 $\boxed{<}$ -3 f. 20 $\boxed{>}$ 5

Addition of Integers

Add integers

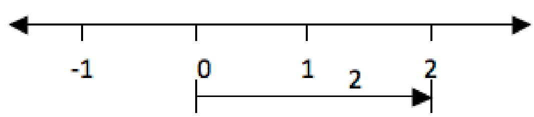
Example 23

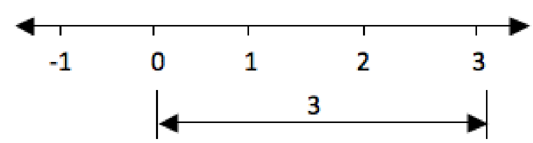
2 + 3

Show a picture of 2 and 3 on a number line.

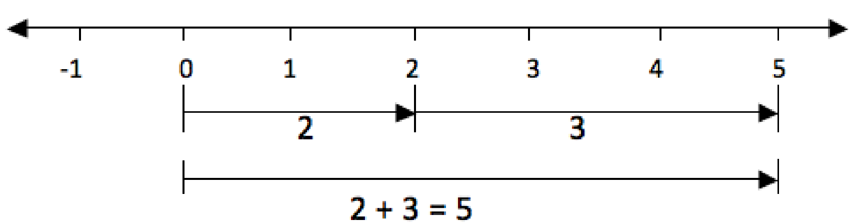
Solution

Show a picture of 2 and 3 on a number line.

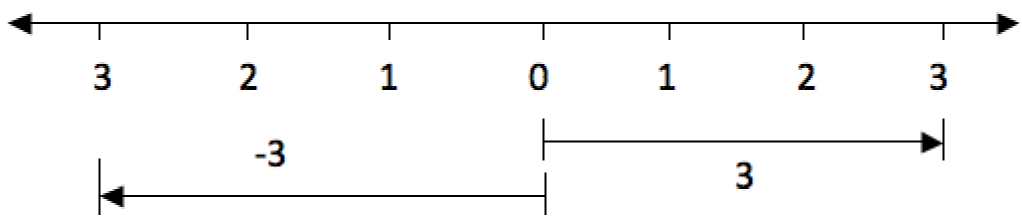




Then add 2 and 3. It will appear as here below



When drawing integers on a number line, the arrows for the positive numbers goes to the right while the arrows for the negative numbers goes to the left. Consider an illustration bellow.

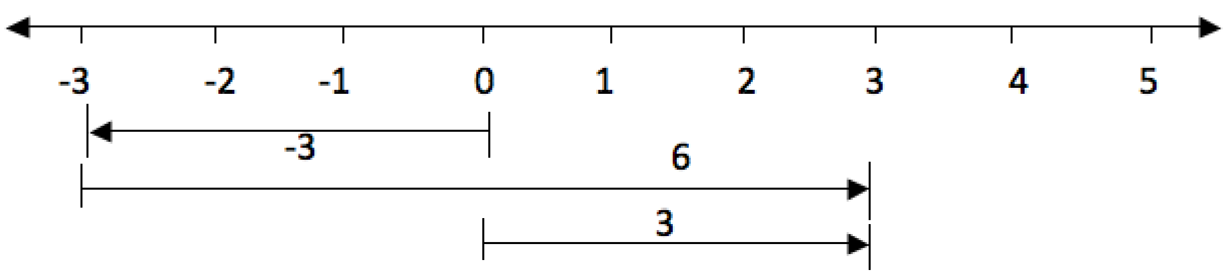


The distance from 0 to 3 is the same as the distance from 0 to -3, only the directions of their arrows differ. The arrow for positive 3 goes to the right while the arrow for the negative 3 goes to the left.

Example 24

-3 + 6

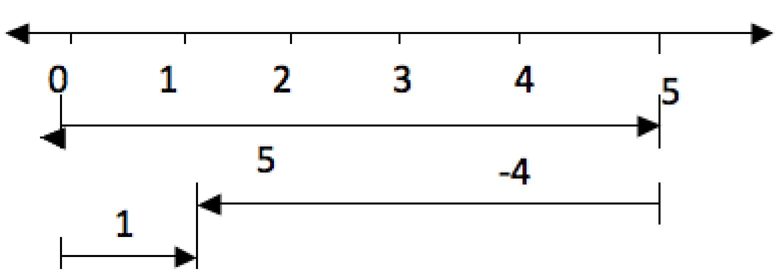
Solution



Subtraction of Integers

Subtract integers

Since subtraction is the opposite of addition, if for example you are given 5-4 is the same as 5 + (-4). So if we have to subtract 4 from 5 we can use a number line in the same way as we did in addition. Therefore 5-4 on a number line will be:



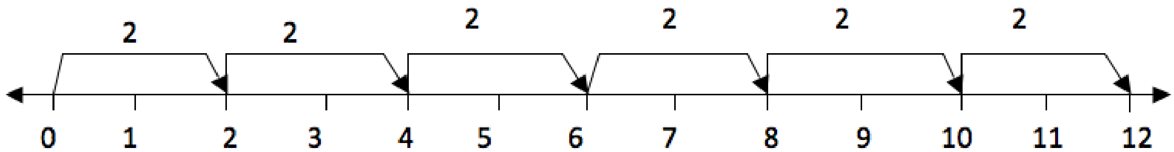
Take five steps from 0 to the right and then four steps to the left from 5. The result is 1.

Multiplication of Integers

Multiply integers

Example 25

2×6 is the same as add 2 six times i.e. 2×6 = 2 + 2 + 2 + 2 + 2 +2 = 12. On a number line will be:



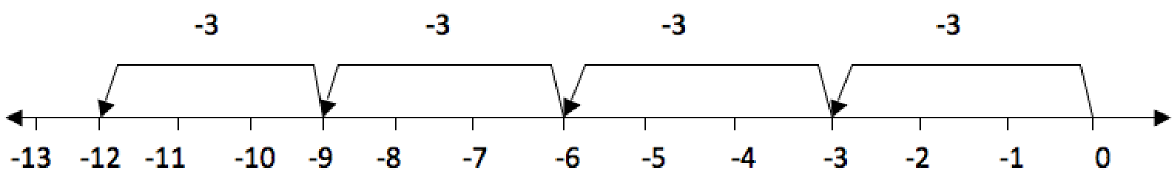
Example 2

-3 × 4

Solution

-3 × 4 = -3 + -3 + -3 + -3 = -12

On a number line will be:



Multiplication of a negative integer by a negative integer cannot be shown on a number line but the product of these two negative integers is a positive integer.

From the above examples we note that multiplication of two positive integers is a positive integer. And multiplication of a positive integer by a negative integer is a negative integer. In summary:

- (+)×,(+) = (+)
- (-)×,(-) = (+)
- (+)×,(-) = (-)
- (-)×,(+) = (-)

Division of Integers

Divide integers

Example 26

6÷3 is the same as saying that, which number when you multiply it by 3 you will get 6, that number is 2, so, 6÷3 = 2.

Therefore division is the opposite of multiplication. From our example 2×3 = 6 and 6÷3 = 2. Thus multiplication and division are opposite to each other.

Dividing two integers which are both positive the quotient (answer) is a positive integer. If they are both negative also the quotient is positive. If one of the integer is positive and the other is negative then the quotient is negative. In summary:

- (+)÷(+) = (+)
- (-)÷(-) = (+)
- (+)÷(-) = (-)
- (-)÷(+) = (-)

Mixed Operations on Integers

Perform mixed operations on integers

You may be given more than one operation on the same problem. Do multiplication and division first and then the rest of the signs. If there are brackets, we first open the brackets and then we do division followed by multiplication, addition and lastly subtraction. In short we call it BODMAS. The same as the one we did on operations on whole numbers.

Example 27

9÷3 + 3×2 -1 =

Solution

9÷3 + 3×2 -1

=3 + 6 -1 (first divide and multiply)

=8 (add and then subtract)

Example 28

(12÷4 -2) + 4 – 7=

Solution

(12÷4 -2) + 4 – 7

=1 + 4 – 7 (do operations inside the brackets and divide first)

=5 – 7 (add)

=2

FRACTIONS

A fraction is a number which is expressed in the form of **a/b** where **a -** is the top number called numerator and **b-** is the bottom number called denominator.

Proper, Improper and Mixed Numbers

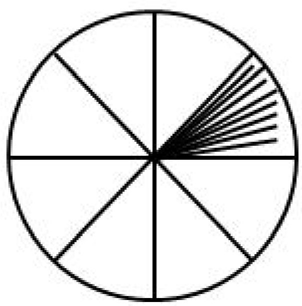
A Fraction

Describe a fraction

A fraction is a number which is expressed in the form of **a/b** where **a -** is the top number called numerator and **b**- is the bottom number called denominator.

Consider the diagram below

The shaded part in the diagram above is 1 out of 8, hence mathematically it is written as 1/8



Example 1

(a) 3 out of 5 (three-fifths) = 3/5

Example 2

(b) 7 0ut of 8 (i.e seven-eighths) = 7/8

Example 3

a. 5/12=(5 X 3)/(12 x 3) =15/36

b. 3/8 =(3 x 2)/(8 X 2) = 6/16

Dividing the numerator and denominator by the same number (This method is used to simplify the fraction)

Difference between Proper, Improper Fractions and Mixed Numbers

Distinguish proper, improper fractions and mixed numbers

Proper fraction -is a fraction in which the numerator is less than denominator

Example 4

4/5, 1/2, 11/13

Improper fraction -is a fraction whose numerator is greater than the denominator

Example 5

12/7, 4/3, 65/56

Mixed fraction -is a fraction which consist of a whole number and a proper fraction

Example 6

$1\frac{2}{9}$, $5\ \frac{35}{203}$, $36\frac{1}{2}$

(a) To convert mixed fractions into improper fractions, use the formula below

(b)To convert improper fractions into mixed fractions, divide the numerator by the denominator

$$\boxed{\frac{\text{denominator of proper fraction x whole number + numerator of proper fraction}}{\text{denominator of proper fraction}}}$$

Example 7

Convert the following mixed numbers into improper fractions

(a) $11\frac{2}{3}$ (b) $6\frac{1}{7}$ (c) $3\frac{7}{8}$ (d) $16\frac{4}{9}$ (e) $6\frac{7}{11}$

Comparison of Fractions

In order to find which fraction is greater than the other, put them over a common denominator, and then the greater fraction is the one with greater numerator.

A Fraction to its Lowest Terms

Simplify a fraction to its lowest terms

Example 8

For the pair of fractions below, find which is greater

$$(a)\ \frac{7}{9},\ \frac{6}{7} \qquad (b)\ \frac{3}{8},\ \frac{1}{3}$$

Solution

$$(a)\ \frac{7}{9} = \frac{7\ x\ 7}{9\ x\ 7} = \frac{49}{63} \qquad \text{and} \qquad \frac{6}{7} = \frac{6\ x\ 9}{7\ x\ 9} = \frac{54}{63}$$

$$\therefore\ \frac{6}{7} \text{ is greater than } \frac{7}{9}$$

$$(b)\ \frac{3}{8} = \frac{3\ x\ 3}{8\ x\ 3} = \frac{9}{24} \qquad \text{and} \qquad \frac{1}{3} = \frac{1\ x\ 8}{3\ x\ 8} = \frac{8}{24}$$

$$\therefore\ \frac{3}{8} \text{ is greater than } \frac{1}{3}$$

Equivalent Fractions

Identify equivalent fractions

Equivalent Fraction

- Are equal fractions written with different denominators
- They are obtained by two methods

(a) **Multiplying the numerator and denominator by the same number**

$$(i)\quad \frac{5}{12} = \frac{5\,x\,3}{12\,x\,3} = \frac{15}{36}$$

$$(ii)\quad \frac{3}{8} = \frac{3\,x\,2}{8\,x\,2} = \frac{6}{16}$$

(a) **Dividing the numerator and denominator by the same number (This method is used to simplify the fraction**

$$(i)\quad \frac{\cancel{14}^{7}}{\cancel{18}^{9}} = \frac{7}{9} \quad \text{(both numerator and denominator are divided by 2)}$$

$$(ii)\quad \frac{\cancel{40}^{5}}{\cancel{88}^{11}} = \frac{5}{11} \quad \text{(both numerator and denominator are divided by 8)}$$

NOTE: The fraction which cannot be simplified more is said to be in its lowest form

Example 9

Simplify the following fractions to their lowest terms

(a) $\frac{25}{40}$ (b) $\frac{26}{36}$ (c) $\frac{13}{39}$ (d) $\frac{100}{80}$ (e) $\frac{45}{120}$

Solution

(a) $\frac{\cancel{25}^{5}}{\cancel{40}^{8}} = \frac{5}{8}$ (each number divided by 5)

(b) $\frac{\cancel{26}^{13}}{\cancel{36}^{18}} = \frac{13}{18}$ (each number divided by 2)

(c) $\frac{\cancel{13}^{1}}{\cancel{39}^{3}} = \frac{1}{3}$ (each number divided by 13)

(d) $\frac{\cancel{100}^{5}}{\cancel{80}^{4}} = \frac{5}{4}$ (each number divided by 20)

(e) $\frac{\cancel{45}^{3}}{\cancel{120}^{8}} = \frac{3}{8}$ (each numberdivided by 15)

Fractions in Order of Size

Arrange fractions in order of size

Example 10

Arrange in order of size, starting with the smallest, the fraction

$\frac{2}{3}, \frac{4}{7}, \frac{3}{8}, \frac{5}{9}$

Solution

Put them over the same denominator, that is find the L.C.M of 3, 7, 8 and 9

Operations and Fractions

Addition of Fractions

Add fractions

Operations on fractions involves **addition, subtraction, multiplication** and **division**

- Addition and subtraction of fractions is done by putting both fractions under the same denominator and then add or subtract
- Multiplication of fractions is done by multiplying the numerator of the first fraction with the numerator of the second fraction, and the denominator of the first fraction with the denominator the second fraction.
- For mixed fractions, convert them first into improper fractions and then multiply
- Division of fractions is done by taking the first fraction and then multiply with the reciprocal of the second fraction
- For mixed fractions, convert them first into improper fractions and then divide

Example 11

Find

(a) $\frac{1}{5} + \frac{1}{4}$ (b) $\frac{5}{7} + \frac{2}{3}$ (c) $\frac{2}{5} + \frac{7}{13}$

Solution

(a) $\frac{1}{5} = \frac{1 \, x \, 4}{5 \, x \, 4} = \frac{4}{20}$ and $\frac{1}{4} = \frac{1 \, x \, 5}{4 \, x \, 5} = \frac{5}{20}$

$\therefore \frac{1}{5} + \frac{1}{4} = \frac{4}{20} + \frac{5}{20} = \frac{9}{20}$

(b) $\frac{5}{7} = \frac{5 \, x \, 3}{7 \, x \, 3} = \frac{15}{21}$ and $\frac{2}{3} = \frac{2 \, x \, 7}{3 \, x \, 7} = \frac{14}{21}$

$\therefore \frac{5}{7} + \frac{2}{3} = \frac{15}{21} + \frac{14}{21} = \frac{29}{21} = 1\frac{8}{21}$

(c) $\frac{2}{5} = \frac{2 \, x \, 13}{5 \, x \, 13} = \frac{26}{65}$ and $\frac{7}{13} = \frac{7 \, x \, 5}{13 \, x \, 5} = \frac{35}{65}$

$\therefore \frac{2}{5} + \frac{7}{13} = \frac{26}{65} + \frac{35}{65} = \frac{61}{65}$

Subtraction of Fractions

Subtract fractions

Example 12

Evaluate

(a) $\frac{2}{3} - \frac{3}{8}$ (b) $\frac{3}{4} - \frac{1}{5}$ (c) $\frac{5}{16} - \frac{1}{8}$

Solution

(a) $\frac{2}{3} = \frac{2 \ x\, 8}{3 \ x \ 8} = \frac{16}{24}$ and $\frac{3}{8} = \frac{3 \ x\, 3}{8 \ x \ 3} = \frac{9}{24}$

$\therefore \frac{2}{3} - \frac{3}{8} = \frac{16}{24} - \frac{9}{24} = \frac{7}{24}$

(b) $\frac{3}{4} = \frac{3 \ x\, 5}{4 \ x \ 5} = \frac{15}{20}$ and $\frac{1}{5} = \frac{1 \ x\, 4}{5 \ x \ 4} = \frac{4}{20}$

$\therefore \frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20} = \frac{11}{20}$

(c) $\frac{5}{16} = \frac{5 \ x\, 8}{16 \ x \ 8} = \frac{40}{128}$ and $\frac{1}{8} = \frac{1 \ x\, 16}{8 \ x \ 16} = \frac{16}{128}$

$\therefore \frac{5}{16} - \frac{1}{8} = \frac{40}{128} - \frac{16}{128} = \frac{24}{128}$

Multiplication of Fractions

Multiply fractions

Example 13

Find (a) $\frac{2}{3} \ x \ \frac{3}{4}$ (b) $\frac{15}{16} \ x \ \frac{24}{25}$ (c) $\frac{3}{11} \ x \ 4$

Solution

(a) $\frac{2}{3} \ x \ \frac{3}{4} = \frac{2 \ x \ 3}{3 \ x \ 4} = \frac{6}{12} = \frac{1}{2}$

(b) $\frac{15}{16} \ x \ \frac{24}{25} = \frac{15 \ x \ 24}{16 \ x \ 25} = \frac{360}{400} = \frac{9}{10}$

(c) $\frac{3}{11} \ x \ 4 = \frac{3}{11} \ x \ \frac{4}{1} = \frac{3 \ x \ 4}{11 \ x \ 1} = \frac{12}{11} = 1\frac{1}{11}$

Division of Fractions

Divide fractions

Example 14

Evaluate (a) $\frac{3}{5} \div \frac{2}{3}$ (b) $\frac{23}{36} \div \frac{45}{48}$ (c) $\frac{5}{7} \div 8$

Solution

(a) $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \; x \; \frac{3}{2} = \frac{3 \; x \; 3}{5 \; x \; 2} = \frac{9}{10}$

(b) $\frac{23}{36} \div \frac{45}{48} = \frac{23}{36} \; x \; \frac{48}{45} = \frac{23 \; x \; 48}{36 \; x \; 45} = \frac{1104}{1520} = \frac{69}{95}$

(c) $\frac{5}{7} \div 8 = \frac{5}{7} \div \frac{8}{1} = \frac{5}{7} \; x \; \frac{1}{8} = \frac{5 \; x \; 1}{7 \; x \; 8} = \frac{5}{56}$

Mixed Operations on Fractions

Perform mixed operations on fractions

Example 15

Find (a) $2\frac{3}{5} + 3\frac{3}{4}$ (b) $9\frac{1}{2} - 4\frac{5}{8}$

Solution

(a) $2\frac{3}{5} = \frac{13}{5} = \frac{13 \; x \, 4}{5 \; x \; 4} = \frac{52}{20}$ and $3\frac{3}{4} = \frac{15}{4} = \frac{15 \; x \, 5}{4 \; x \; 5} = \frac{75}{20}$

$\therefore \; 2\frac{3}{5} + 3\frac{3}{4} = \frac{52}{20} + \frac{75}{20} = \frac{127}{20} = 6\frac{7}{20}$

(b) $9\frac{1}{2} = \frac{19}{2} = \frac{19 \; x \, 8}{2 \; x \; 8} = \frac{152}{16}$ and $4\frac{5}{8} = \frac{37}{8} = \frac{37 \; x \; 2}{8 \; x \; 2} = \frac{74}{16}$

$\therefore \; 9\frac{1}{2} - 4\frac{5}{8} = \frac{152}{16} - \frac{74}{16} = \frac{78}{16} = 4\frac{14}{16}$

Example 16

Evaluate (a) $5\frac{2}{3} \div 1\frac{1}{2}$ (b) $4\frac{3}{5}\ x\ 7\frac{1}{8}$

Solution

(a) $5\frac{2}{3} \div 1\frac{1}{2} = \frac{17}{3} \div \frac{3}{2} = \frac{17}{3}\ x\ \frac{2}{3} = \frac{17\ x\ 2}{3\ x\ 3} = \frac{34}{9} = 3\frac{7}{9}$

(b) $4\frac{3}{5}\ x\ 7\frac{1}{8} = \frac{23}{5}\ x\ \frac{57}{8} = \frac{23\ x\ 57}{5\ x\ 8} = \frac{1311}{40} = 32\frac{31}{40}$

Word Problems Involving Fractions

Solve word problems involving fractions

Example 17

1. Musa is years old. His father is 3¾times as old as he is. How old is his father?
2. 1¾of a material are needed to make suit. How many suits can be made from

DECIMAL AND PERCENTAGE

Decimals

The Concept of Decimals

Explain the concept of decimals

A decimal- is defined as a number which consist of two parts separated by a point.The parts are whole number part and fractional part

Example 1

$$5.6\ ; \quad 5 = \text{whole number part} \quad \text{and}$$

$$0.6 = \text{fractional part i. e it can be written as } \frac{6}{10}$$

Therefore $5.6 = 5\frac{6}{10}$

Example 2

(i) $13.35 = 13\frac{35}{100}$

(ii) $0.123 = \frac{123}{1000}$

(iii) $2.08 = 2\frac{8}{100}$

Conversion of Fractions to Terminating Decimals and Vice Versa

Convert fractions to terminating decimals and vice versa

The first place after the decimal point is called tenths.The second place after the decimal point is called hundredths e.t.c

Consider the decimal number 8.152

The place value of 1 – is tenths $\left(\text{i. e } \frac{1}{10}\right)$, 5 – is hundredths $\left(\text{i. e } \frac{5}{100}\right)$ and 2 – is thousandths $\left(\text{i. e } \frac{2}{1000}\right)$

NOTE

- To convert a fraction into decimal, divide the numerator by denominator
- To convert a decimal into fraction, write the digits after the decimal point as tenths, or hundredths or thousandths depending on the number of decimal places.

Example 3

Convert the following fractions into decimals

(a) $\frac{1}{10}$ (b) $\frac{27}{100}$ (c) $8\frac{2}{5}$ (d) $\frac{37}{125}$ (e) $\frac{53}{250}$

Solution

Divide the numerator by denominator

(a)

$$\begin{array}{r} 0.1 \\ 10\overline{)\,1} \\ \underline{-0} \\ 10 \\ \underline{-10} \end{array}$$

$\therefore \frac{1}{10} = 0.1$

(b)

$$\begin{array}{r} 0.27 \\ 100\overline{)\,27} \\ \underline{-0} \\ 270 \\ \underline{-200} \\ 700 \\ \underline{-700} \end{array}$$

$\therefore \frac{27}{100} = 0.27$

Operations and Decimals

Addition of Decimals

Add decimals

Example 4

Evaluate

(a) $28.3 + 4.96 + 110.9$

(b) $23.4 + 67.98$

Solution

(a)
$$\begin{array}{r} 28.30 \\ 4.96 \\ +\ \underline{110.90} \\ \underline{144.16} \end{array}$$

(b)
$$\begin{array}{r} 23.40 \\ +\ \underline{67.98} \\ \underline{91.38} \end{array}$$

Subtraction of Decimals

Subtract decimals

Example 5

(a) $7.4 - 3.9$

(b) $12.8 - 17.2$

Solution

(a)
$$\begin{array}{r} 7.4 \\ -\ \underline{3.9} \\ \underline{3.3} \end{array}$$

(b)
$$\begin{array}{r} 120.8 \\ -\ \underline{17.2} \\ \underline{103.6} \end{array}$$

Multiplication of Decimals

Multiply decimals

Example 6

(a) $23.1 \ x \ 6.7$

(b) $4.8 \ x \ 67.98$

Solution

(a) $23.1 \ x \ 6.7$

$$231 \ x \ 67 = 15477$$

$$\therefore \quad 23.1 \ x \ 6.7 = 154.77$$

(b) $4.8 \ x \ 67.98$

$$48 \ x \ 6798 = 326304$$

$$\therefore \quad 4.8 \ x \ 67.98 = 326.304$$

Mixed Operations with Decimals

Perform mixed operations with decimals

Example 7

(a) $4.5 \div 0.2$

(b) $732 \div 1.28$

Solution

(a) $4.5 \div 0.2 = \frac{4.5}{0.2} = \frac{4.5 \ x \ 10}{0.2 \ x \ 10} = \frac{45}{2} = 22.5$

(b) $732 \div 1.28 = \frac{732}{1.28} = \frac{732 \ x \ 100}{1.28 \ x \ 100} = \frac{73200}{128} = 571.875$

Word Problems Involving Decimals

Solve word problems involving decimals

Example 8

If 58 out of 100 students in a school are boys, then write a decimal for the part of the school that consists of boys.

Solution

We can write a fraction and a decimal for the part of the school that consists of boys.

Fraction	**decimal**
58/100	0.58

Percentages

Expressing a Quantity as a Percentage

Express a quantity as a percentage

The percentage of a quantity is found by converting the percentage to a fraction or decimal and then multiply it by the quantity.

Example 9

Find (a) 25 % of Tsh 60,000

(b) $12\frac{1}{2}$ % of 80 cm^2

(c) 1.2 % of 120 minutes

Solution

(a) 25 % of Tsh 60,000 $= \frac{25}{100} \; x \; 60{,}000 = \frac{25 \; x \; 60{,}000}{100} = \frac{1\,500\,000}{100} =$ Tsh 15 000

(b) $12\frac{1}{2}$ % of 80 $cm^2 = \frac{25}{2}$ % x 80 $cm^2 = \frac{25}{200} \; x$ 80 $cm^2 = \frac{25 \; x \, 80 \; cm^2}{200} = \frac{2000}{200}$ cm^2 = 100 cm^2

(c) 1.2 % of 120 minutes $= \frac{1.2}{100} \; x$ 120 minutes $= \frac{1.2 \; x \; 120}{100} = \frac{144}{100} =$ 1.44 minutes

NOTE: The concept of percentage of a quantity can be used to solve the problems involving percentage increase and decrease as shown in the below examples:-

A Fractions into Percentage and Vice Versa

Convert a fraction into percentage and vice versa

To change a fraction or a decimal into a percentage, multiply it by 100%

Example 10

Convert the following fractions into percentages

(a) $\frac{9}{40}$

(b) $\frac{1}{12}$

(c) $2\frac{1}{2}$

(d) $\frac{7}{8}$

Solution

(a) $\frac{9}{40}\ x\ 100\% = \frac{9\ x\ 100}{40}\% = \frac{900}{40}\ \% = 22.5\ \%$

(b) $\frac{1}{12}\ x\ 100\% = \frac{1\ x\ 100}{12}\% = \frac{100}{12}\ \% = 8.3\ \%$

(c) $2\frac{1}{2}\ x\ 100\% = \frac{5}{2}\ x\ 100\ \% = \frac{5\ x\ 100}{2}\% = \frac{500}{2}\ \% = 225\ \%$

(d) $\frac{7}{8}\ x\ 100\% = \frac{7\ x\ 100}{8}\ \% = \frac{700}{8}\ \% = 87.5\ \%$

A Decimal into Percentage and Vice Versa

Convert a decimal into percentage and vice versa

To change a percentage into a fraction or a decimal, divide it by 100%

Example 11

Convert the following percentages into decimals

(a) $66\frac{2}{3}$ %

(b) 48.5 %

(c) $12\frac{1}{2}$ %

(d) 150 %

Solution

(a) $66\frac{1}{2}\ \% = \frac{133}{2}\ \% = \frac{133}{2} \div 100 = \frac{133}{2}\ x\ \frac{1}{100} = \frac{133}{200}$

(b) $48.5\ \% = \frac{48.5}{100} = \frac{485}{1000} = \frac{97}{200}$

(c) $12\frac{1}{2}\ \% = \frac{25}{2}\ \% = \frac{25}{2} \div 100 = \frac{25}{2}\ x\ \frac{1}{100} = \frac{25}{200} = \frac{1}{8}$

(d) $150\ \% = \frac{150}{100} = \frac{3}{2}$

Percentages in Daily Life

Apply percentages in daily life

Example 12

In an assignment, Regina scored 9 marks out of 12. Express this as a percentage

Solution

$$9 \text{ out of } 12 = \frac{9}{12}$$

$$\text{Percentage} = \frac{9}{12}\ x\ \ 100\ \% = \frac{9\ x\ 100}{12}\ \% = \frac{900}{12}\ \% = 75\ \%$$

Example 13

A school has 400 students of which 250 are girls. What percentage of the students are not girls?

Solution

$$\text{Percentage of girls} = \frac{250}{400}\ x\ \ 100\ \% = \frac{250\ x\ 100}{400}\ \% = \frac{25000}{400}\ \% = 62.5\ \%$$

$$\text{Percentage which are not girls} = 100\ \% - 62.5\ \% = 37.5\ \%$$

UNITS

A unit – is defined as a symbol or sign which is assigned to a number to describe a kind of measurement made

Units of Length

Convertion of One Unit of Length to Another

Convert one unit of length to another

The conversion of one unit to another is done by considering the arrangement below

km						1
hm					1	0
dam				1	0	0
m			1	0	0	0
dm		1	0	0	0	0
cm	1	0	0	0	0	0
mm	0	0	0	0	0	0

Example, from the above we get

$$1\text{ km} = 1{,}000{,}000\text{ mm}$$
$$1\text{ hm} = 100{,}000\text{ mm}$$
$$1\text{ dam} = 1000\text{ cm}$$
$$1\text{ m} = 100\text{ cm}$$

Computations on Metric Units of Length

Perform computations on metric units of length

Example 1

Convert

(a) 12cm to dm

(b) $2{,}500\text{ mm}$ to m

(c) 87 km to cm

Solution

(a) 12cm to dm

$$1dm = 10\ cm$$
$$? = 12\ cm$$

By cross multiplication, we get

$$= \frac{1\ dm\ \ x\ \ 12\ cm}{10\ cm} = \frac{12}{10}\ dm = 1.2\ dm$$

(b) 2,500 mm to m

$$1m = 1000\ mm$$
$$? = 2500\ mm$$

By cross multiplication, we get

$$= \frac{1\ m\ \ x\ \ 2\ 500\ mm}{1000\ mm} = \frac{2500}{1000}\ m = 2.5\ m$$

(c) 87 km to cm

$$1\ km = 100\ 000\ cm$$
$$87\ km = \ ?$$

By cross multiplication, we get

$$= \frac{87\ km\ \ x\ \ 100\ 000\ cm}{1\ km} = \frac{8\ 700\ 000}{1}\ cm = 8\ 700\ 000\ cm$$

Unit of Mass

Convertion of One Unit of Mass to Another

Convert one unit of mass to another

The conversion of one unit to another is done by considering the arrangement below

kg						1
hg					1	0
dag				1	0	0
g			1	0	0	0
dg		1	0	0	0	0
cg	1	0	0	0	0	0
mg	0	0	0	0	0	0

Example, from the above

$$1\text{ kg} = 1{,}000{,}000\text{ mg}$$
$$1\text{ hg} = 100{,}000\text{ mg}$$
$$1\text{ dag} = 1000\text{ cg}$$
$$1\text{ g} = 100\text{ cg}$$

The conversion of *tonne*to other units is done converting it to *kilogram*first and then from*kilogram*to the required unit.

1 tonne = 1000kg

Computation on Metric Units of Mass

Perform computation on metric units of mass

Example 2

Convert (i) 8 500 g to kg

(ii) 0.000025 kg to mg

(iii) 4.67cg to dg

Solution

(i) 8 500 g to kg

$$1\ kg = 1000\ g$$
$$?\ = 8\,500\ g$$
$$= \frac{1\ kg\ \ x\,8\,500\ g}{1000\ g} = \frac{8\,500}{1000}\ kg = 8.5\ kg$$

(ii) 0.000025 kg to mg

$$1\ kg = 1\,000\,000\ mg$$
$$0.000025\ kg = \ ?$$
$$= \frac{0.000025\ kg\ \ x\ 1\,000\,000\ mg}{1\ kg} = \frac{25}{1}\ mg = 25\ mg$$

(iii) 4.67cg to dg

$$1\ dg = 10\ cg$$
$$?\ = 4.67\ cg$$
$$= \frac{1\ dg\ \ x\ 4.67\ cg}{10\ cg} = \frac{4.67}{10}\ dg = 0.467\ dg$$

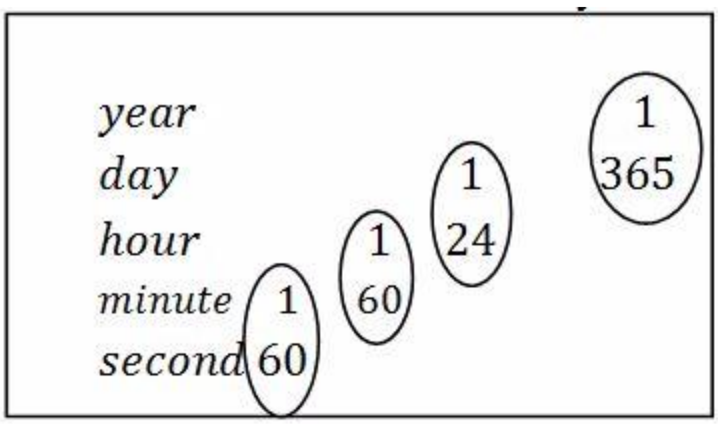
Units of Time

The units of time are of two types, smaller and larger units of time.Smaller units of time includes seconds, minutes, hours and days.Larger units of time includes week, month, year, decade, century,millennium.

Convertion of One Unit of Time to another

Convert one unit of time to another

The conversion of units of time to another can be done by considering the arrangement below



Example, from the circle above

$1\ min = 60\ sec \qquad 1\ hour = 60\ \text{min} \qquad 1\ day = 24\ hours \qquad 1\ year = 365\ days$

Also

$$1\ \text{year} = 365 \quad x\ 24 \quad x \quad 60 \quad x\ 60\ \text{seconds}$$

$$1\ \text{days} = 24\ x \quad 60\ x \quad 60\ \text{seconds}$$

$$1\ \text{hour} = 60\ x \quad 60\ \text{seconds}$$

$$1\ \text{minute} = 60\ \text{seconds}$$

Example 3

Convert

(i) $54{,}000$ seconds to minutes

(ii) $7{,}200$ minutes to hours

(iii) 6 hours to seconds

Solution

(i) 54, 000 seconds to minutes

$$1\ min = 60\ sec$$
$$?\quad = 54\ 000\ sec$$
$$= \frac{1\ min\ x\ \ 54\ 000\ sec}{60\ sec} = \frac{54\ 000}{60}\ min = 900\ minutes$$

(ii) 7, 200 minutes to hours

$$1\ hour = 60\ minutes$$
$$?\quad = 7\ 200\ minutes$$
$$= \frac{1\ hour\ x\ \ 7\ 200\ minutes}{60\ minutes} = \frac{7\ 200}{60}\ hours = 120\ hours$$

(iii) 6 hours to seconds

$$1\ hour = 60\ x\ 60\ sec$$
$$6\ hours = \quad ?$$
$$= \frac{1\ hours\ x\ \ 60\ x\ 60\ sec}{1\ hour} = \frac{3600}{1}\ seconds = 3\ 600\ seconds$$

Convertion of Unit Time of 12 Hour Clock to 24 Hour Clock and Vice Versa

Read and convert unit time of 12 hour clock to 24 hour clock and vice versa

The hours can exist in two systems: *12- hour clock and 24 - hour clock.*

A 12- *hour clock* has 12 hours between midnight and midday*(a.m)*and 12 hours between midday and midnight*(p.m).*

A 21 - hour has 24 hours in a day.Times in the morning are the same in both systems.For times in the afternoon, convert by adding or subtracting 12 hours

Example 4

Convert the following times from the 12 - hour clock to 24 - hour clock.

(i) $5.30\ a.m$

(ii) $1.40\ p.m$

(iii) $7.15\ \ p.m$

Solution

(i) $5.30\ a.m = 0530\ hrs$

(ii) $1.40\ p.m = (0140 + 1200) = 1340\ hrs$

(iii) $7.15\ \ p.m = (0715 + 1200) = 1915\ hrs$

Example 5

Convert the following times from the 24 - hour clock to 12 - hour clock.

(i) $0450\ hrs$

(ii) $1245\ hrs$

(iii) $2300\ \ hrs$

Solution

(i) $0450\ hrs = 4.50\ a.m$

(ii) $1245\ hrs = 12.45\ p.m$

(iii) $2300\ \ hrs = (2300 - 1200) = 11.00\ p.m$

Units of Capacity

Standard Unit of Measuring Capacity

State the standard unit of measuring capacity

Capacity is related to the volume.

Definitions:

- Capacity-is defined as the ability hold or contain something
- The S.I unit of capacity is *litre.*
- Volume –is defined as the amount of space occupied by a substance
- The S.I unit of volume is *cubic metres* (m^3)

Capacity is related to the volume as follows:

$1\ litre = 1000cm^3 = 0.001m^3 = 1dm^3$

Also $1\ ml = 1\ cm^3$

Other units related to litre are kiloliter (kl), hectoliter (hl), decalitre (dal), litre (l), deciliter (dl), centiliter (cl) and mililitre (ml).

The conversion of one unit to another is done by considering the arrangement below

kl						1
hl					1	0
dal				1	0	0
l			1	0	0	0
dl		1	0	0	0	0
cl	1	0	0	0	0	0
ml	0	0	0	0	0	0

Example, from the above

$$1\,kl = 1{,}000{,}000\,ml$$
$$1\,hl = 100{,}000\,ml$$
$$1\,dal = 1000\,cl$$
$$1\,l = 100\,cl$$

The Litre in Daily Life

Use the litre in daily life

Example 6

Convert the following units into

(i) $3500\,ml$

(ii) $0.006\ m^3$

(iii) $4000\ dm^3$

(iv) $500\,mm^3$

Solution

(i) Convert first $3500\, ml\ \ to\ \ cm^3$

$$1\, ml = 1\, cm^3$$

$$3500\, ml = \ \ ?$$

$$= \frac{3500\, ml\ \ x\ \ 1\, cm^3}{1\, ml} = \frac{3500}{1}\ cm^3 = \ 50$$

Then $3\,500 cm^3\ \ to\ \ litres$

$$1\ litre = 1000\, cm^3$$

$$?\qquad = 3\,500 cm^3$$

$$= \frac{3500\, cm^3\ \ x\ \ 1\, litre}{1000\, cm^3} = \frac{3500}{1000}\ litres = \ 3.$$

(ii) $0.006\ \ m^3\ \ to\ \ litres$

$$1\ litre = 0.001\, m^3$$

$$?\qquad = 0.006\ \ m^3$$

$$= \frac{1\, litre\ \ x\ \ 0.006\ \ m^3}{0.001\, m^3} = \frac{0.006}{0.001}\ litres = 6$$

(iii) Convert first $4000\ \ dm^3$ to cm^3

$$1\, dm = 10\, cm$$

$$(1\, dm)^3 = (10\, cm)^3 \ \ \rightarrow \ \ 1\, dm^3 = 1000\, cm^3$$

So

$$1\, dm^3 = 1000\, cm^3$$

$$4000\ \ dm^3 = \ ?$$

$$= \frac{4000\ \ dm^3\ \ x\ \ 1000\, cm^3}{1\, dm^3} = \frac{4\,000\,000}{1}\ \ cm^3 = 4\,000\,000 cm^3$$

Second, convert $4\,000\,000 cm^3\ \ to\ \ litres$

$$1\ litre = 1000\, cm^3$$

$$?\qquad = 4\,000\,000 cm^3$$

$$= \frac{1\, litre\ \ x\ \ 4\,000\,000 cm^3}{1000\, cm^3} = \frac{4\,000\,000}{1000}\ litres = 4\,000\, litres$$

(iv) $500\ mm^3$

Convert first $500\ mm^3$ to cm^3

$$1\ cm = 10\ mm$$

$$(1\ cm)^3 = (10\ mm)^3 \rightarrow 1\ cm^3 = 1000\ mm^3$$

So

$$1\ cm^3 = 1000\ mm^3$$

$$? = 500\ mm^3$$

$$= \frac{1\ cm^3\ x\ 500\ mm^3}{1000\ mm^3} = \frac{500}{1000}\ cm^3 = 0.5\ cm^3$$

Second, convert $0.5\ cm^3\ to\ litres$

$$1\ litre = 1000\ cm^3$$

$$? = 0.5\ cm^3$$

$$= \frac{1\ litre\ x\ 0.5\ cm^3}{1000\ cm^3} = \frac{0.5}{1000}\ litres = 0.0005\ litres$$

APPROXIMATIONS

· Measurements can be rounded to a certain number of significant figures.Approximation- is a process of rounding numbers to a certain degree of accuracy.A number can be rounded to a certain required place value such as to the nearest ten, hundred, and thousand

Rounding Off Numbers

Rounding off Whole Numbers to Given Place Values

Round off whole numbers to given place values

STEPS

- When rounding a number, stand at the digit of the required place value, then look at the next digit to the right; if it is or more, round up (i.e, increase the digit of the required place value by 1) and if it is or less, do not change the digit of the required place value

- Replace all the remaining digits to the right of the required place value with the zeros

Example 1

The population of Tanzania in a census of 2002 was 42,850,671. Round this to the nearest

a. million

b. ten million

Solution

a. The million digit is 2, since the next digit to the right is greater than 5, then we can increase 2 by 1 and put the remaining digits to the right of 2 zeros. There fore;42,850,671 ≈ 43,000,000

b. The ten million digit is 4, since the next digit to the right is less than 5, then we do not change 4 but we put the remaining digits to the right of 4 zeros. There fore;42,850,671 ≈ 40,000,000

Decimals to a Given Number of Decimal Place

Round off decimals to a given number of decimal place

STEPS

- When rounding a decimal, stand at the digit of the required decimal place, then look at the next digit to the right; if it is or more, round up (i.e increase the digit of the required decimal by 1) and if it is or less, do not change the digit of the required decimal place

- Replace all the remaining digits to the right of the required decimal place with the zeros

NOTE

- The first digit after the decimal point is the first decimal place, the second digit after the decimal point is the second decimal place e.t.c

- Example 0.568 is the first decimal place and is the second decimal place

Example 2

Round 0.24736 to the nearest

a. 1 decimal place

b. 2 decimal places

c. 3 decimal places

Solution

a. 0.24736≈ 0.02 (1 d.p)

b. 0.24736≈ 0.025 (2 d.p)

c. 0.24736≈ 0.0247 (1 d.p)

Significant Figures

Significant figures of a number - are the significant digits, counted from left of the number.The first significant figure must be non-zero; following significant figures may take any value.

A Number to a Given Number of Significant Figures

Write a number to a given number of significant figures

Is the number of significant digits including 0 if it is between the first and the last

Examples

a. 13 – has two significant figures

b. 709.43 – has five significant figures

c. 0.0004001 – has four significant figures

STEPS

- When rounding a number to a certain significant figure, stand at the digit of the required significant figure, then look at the next digit to the right; if it is or more, round up (i.e increase the digit of the required significant figure by 1) and if it is or less, do not change the digit of the required significant figure
- Replace all the remaining digits to the right of the required significant figure with the zeros

Example 3

Given the number 45.274 round to

a. 1 first significant figure

b. 2 significant figure

c. significant figure

Solution

a. 50

b. 45

c. 45.3

Example 4

Round 146 400 to

a. 2 first significant figure

b. 4 significant figure

c. 3 significant figure

Solution

a. 150 000

b. 146 400

c. 146 000

Approximations in Calculations

The Knowledge of Rounding Off of Numbers in Computations Involving Large Numbers and Small Numbers

Use the knowledge of rounding off of numbers in computations involving large numbers and small numbers

Approximation can be used in operation to check whether a calculation is correct or not, i.e in addition, subtraction, multiplication and divisione.g 446 x 45 = 20 070

The above calculation can be checked quickly whether it is correct or not by rounding each number to 1 significant figure and then multiply i.e 400 x 50 = 20 000

Therefore, the approximation of 20 000 is close to 20 0070 is correct.Before carrying an operation, each number in a calculation is rounded to 1 significant figure

Example 5

Find the approximate value of

(a) $22.1 + 4.77$

(b) $127 - 79$

(c) $0.53\ x\ 0.68$

(d) $423 \div 19$

(e) $535 \div 1\,121$

Solution

(a) $22.1 + 4.77 \approx 20 + 5 = 25$

(b) $127 - 79 \approx 100 - 80 = 20$

(c) $0.53\ x\ 0.68 \approx 0.5\ x\ 0.7 = 0.35$

(d) $423 \div 19 \approx 400 \div 20 = 20$

(e) $535 \div 1\,121 \approx 500 \div 1000 = 0.5$

Example 6

A school trip of 32 people went to a tour, which costs a transport fee of 580/- each people. What was the approximate total transport cost?

Solution

$32 \times 580 \approx 30 \times 600 = 18\ 000$

The approximate transport cost was 18 000/-

GEOMETRY

Points and Lines

The Concept of a Point

Explain the concept of a point

A **point** – is a smallest geometric figure which gives a position of object in a plane

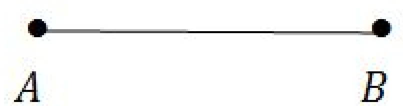
A line segment – is a straight line joining two points in a plane

The Concept of a Point to Draw a Line

Extend the concept of a point to draw a line

A line segment – is a straight line joining two points in a plane

$\overline{AB}$



A line passing through two points e.g A and B and extends without end *(i.e infinitely)* in both directions is denoted by

$\overleftrightarrow{AB}$

The Difference Between a Line, a Line Segment and a Ray

Distinguish between a line, a line segment and a ray

A ray - is a line starting from a point, say A and pass through a point, say B and extends without end in one direction.It is denoted by

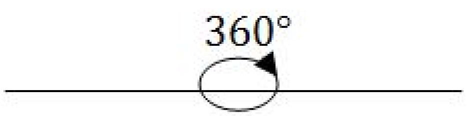
$\overrightarrow{AB}$

Angles and Lines

Angles

Draw angles

An angle – is a measure of an amount of turn. For instance, a complete turn has an angle of 360º

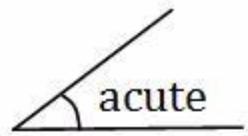


Measuring Angles of Different Size Using a Protractor

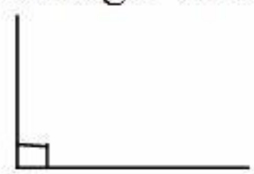
Measure angles of different size using a protractor

There are several types of angles including:- acute, right, complementary, obtuse, supplementary and reflex angle

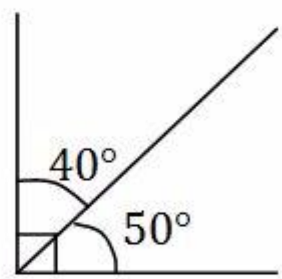
(a) An acute angle – is an angle less than $90°$



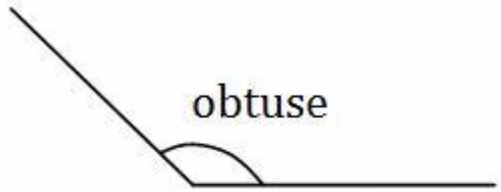
(b) A right angle – is an angle of $90°$



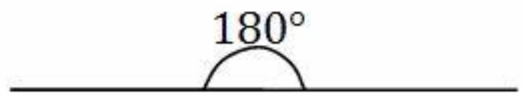
(c) Complementary angles – are angles whose sum is $90°$
e.g $40°$ and $50°$ are complementary angles



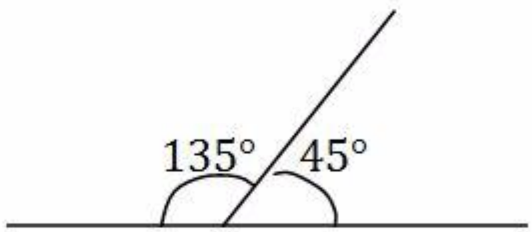
(d) An obtuse angle – is an angle between $90°$ and $180°$



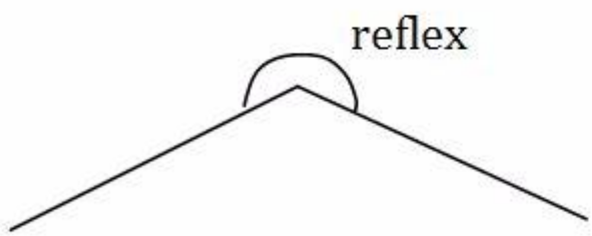
(e) A straight angle - is an angle of $180°$



(f) Supplementary angle – are angles whose sum is $180°$
e.g $135°$ and $45°$ are complementary angles



(g) A reflex angle - is an angle between $180°$ and $360°$



Example 1

a. Two angles are supplementary. One angle is three times the other. What are the angles?

b. Two angles are complementary. One angle is 40º greater than the other. What are the angles?

Solution

(a) Let one angle be x, the other angle is $3x$

Then $x + 3x = 180°$

$$4x = 180°, \quad x = \frac{180°}{4} = 45°$$

$$first\ angle, x = 45°, \quad second\ angle,\ 3x = 3(4$$

$\therefore$ The angles are $45°$ and $135°$

(b) Let one angle be x, the other angle is $x + 40°$

Then $x + (x + 40°) = 90°$

$$x + x + 40° = 90°$$

$$2x + 40° = 90°$$

$$2x = 90° - 40°$$

$$2x = 50°, \quad x = \frac{50°}{2} = 25°$$

$$first\ angle, x = 25°, \quad second\ angle,\ x + 40° = 2$$

$\therefore$ The angles are $25°$ and $65°$

Drawing Angles Using a Protractor

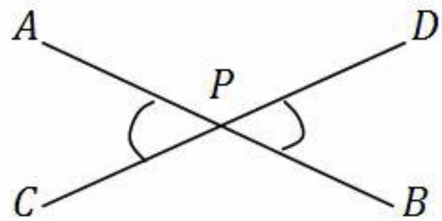
Draw angles using a protractor

The angles formed by crossing lines includes vertically opposite angles, alternate angle and corresponding angles

Vertically opposite angles

The angles on the opposite sides of the crossing lines are equal

Consider two line segments $\overline{AB}$ and $\overline{CD}$ crossing each other

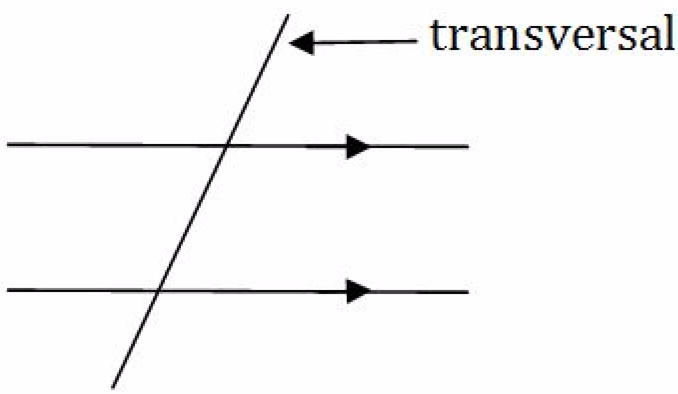


$$\boxed{\begin{array}{ll} \angle APC = \angle BPD & \text{(vertically opposite)} \\ \angle APD = \angle BPC & \text{(vertically opposite)} \end{array}}$$

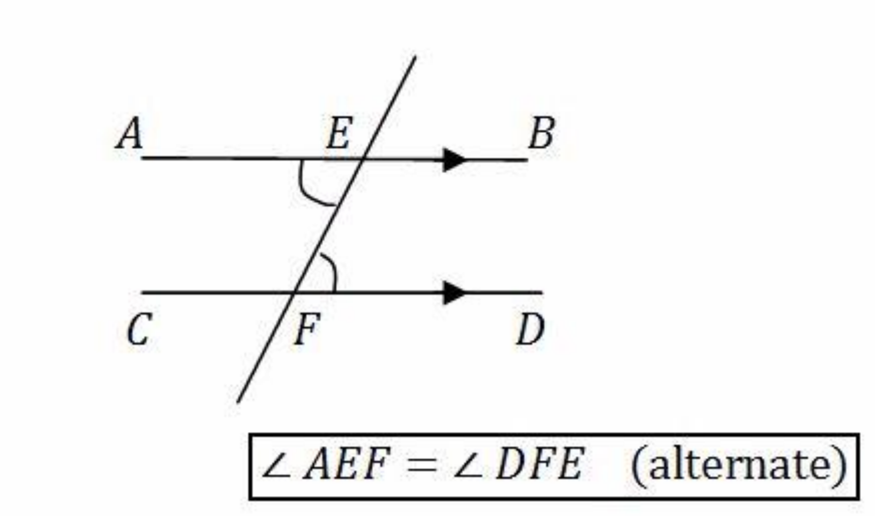
- They are also called $X -$ angles

Alternate angles

Consider a line segment crossing two parallel line segments. This line is called a **transversal**



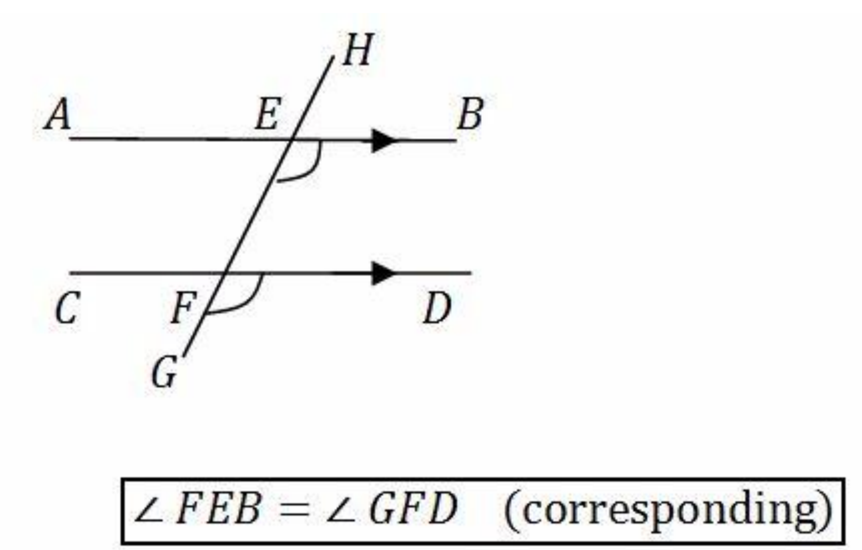
The angles within the parallel line segments on the opposite sides of the transversal are equal



They are also called Z - *angles*

Corresponding angles

The angles on the same side of the transversal and on the same side of the parallel lines are equal.They are called corresponding angles and sometimes called *F - angles*



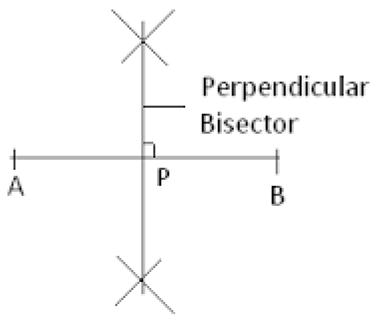
There are also three other pairs of corresponding angles in the diagram above.When showing that two angles are equal you must give reason whether they are vertically opposite, or alternate or corresponding angles.

Constructions

Construction of a Perpendicular Bisector to a Line Segment

Construct a perpendicular bisector to a line segment

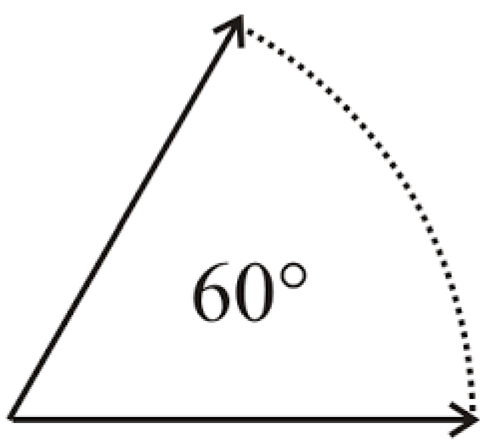
Perpendicular Bisector to a Line Segment is shown below



Contruction of an Angle of 60° Using a Pair of Compasses

Construct an angle of 60° using a pair of compasses

Angle of 60°



Bisection of a Given Angle

Bisect a given angle

Activity 1

Bisect a given angle

Copying a Given Angle by Construction

Copy a given angle by construction

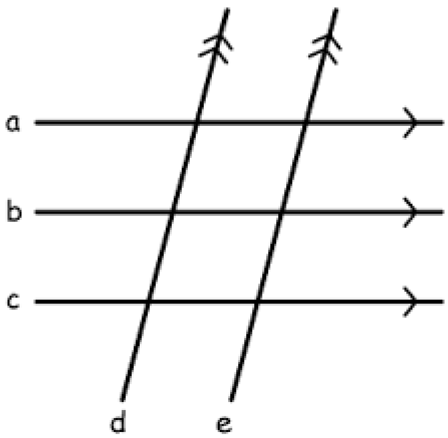
Activity 2

Copy a given angle by construction

Parallel Lines

Construct parallel lines

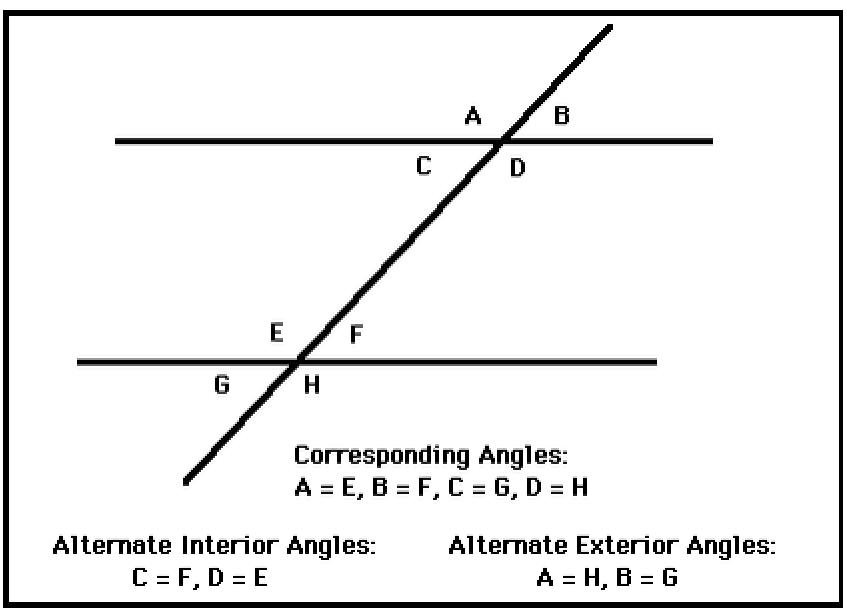
Parallel lines can be shown as below:



Different Types of Angles Formed by Parallel Lines and a Transversal

Identify different types of angles formed by parallel lines and a transversal

Different types of angles are shown below.



Polygons And Regions

A Polygon and a Region

Describe a polygon and a region

A**polygon**is a plane figure whose sides are three or more coplanar segments that intersect only at their endpoints. Consecutive sides cannot be collinear and no more than two sides can meet at any one vertex.

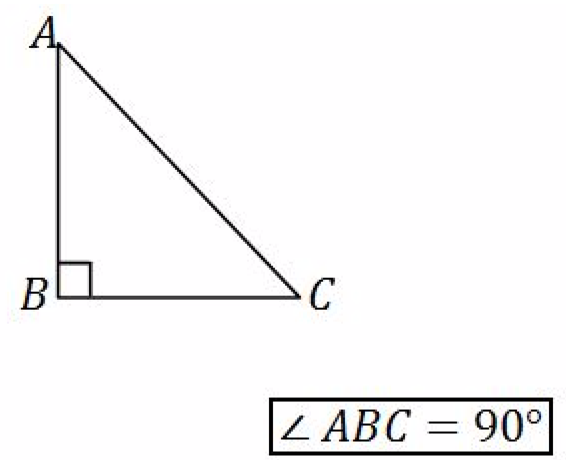
A**polygonal region**is defined as a polygon and its interior.

Different Types of Triangles

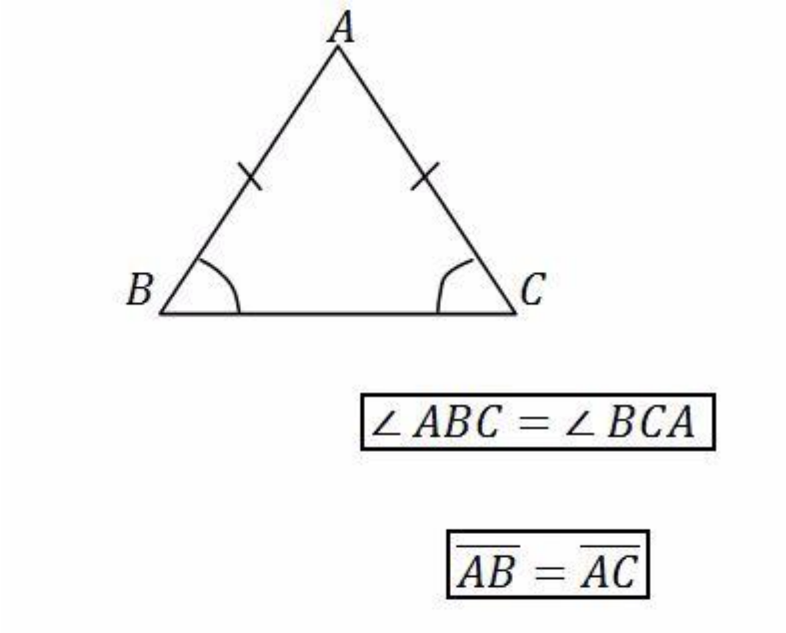
Construct different types of triangles

A triangle – is a polygon with three sides.The sides connect the points called vertices

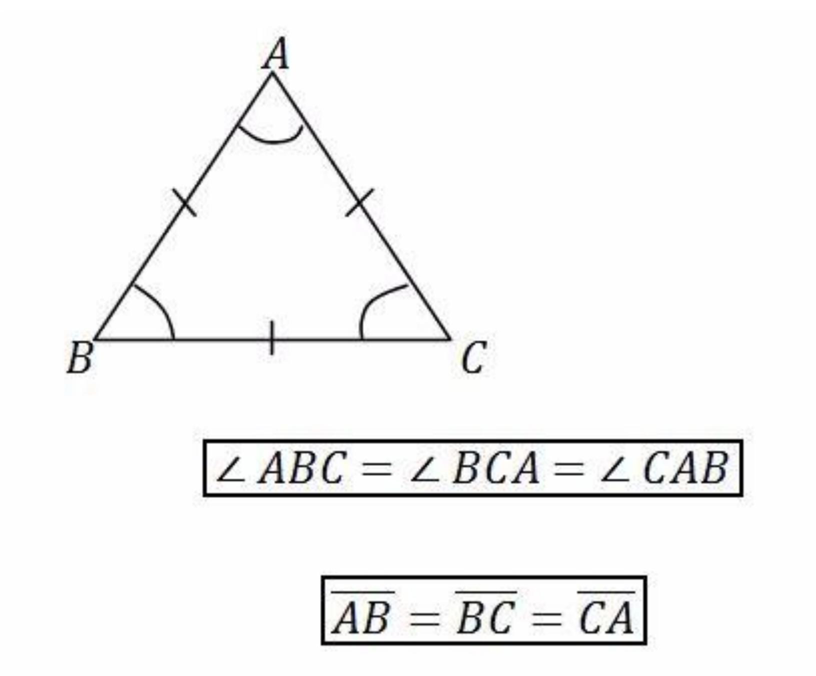
A **right – angled triangle** – has one angle equal to 90º



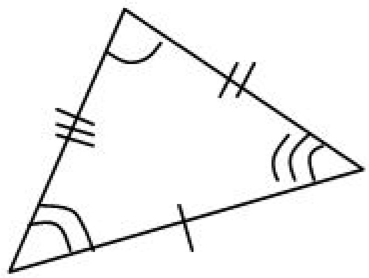
An **isosceles triangle** – has two equal sides and two equal angles



An **equilateral triangle** – has three equal sides and all angles equal



NOTE:A triangle with all sides different and all angles different is called scalene triangle



A triangle with vertices *A*, *B* and *C* is denoted as

$$\Delta\, ABC$$

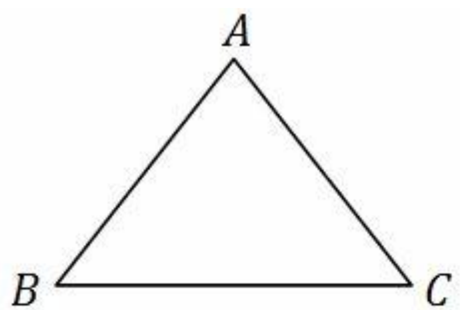
A triangle has two kinds of angles

a. Interior angles

b. Exterior angles

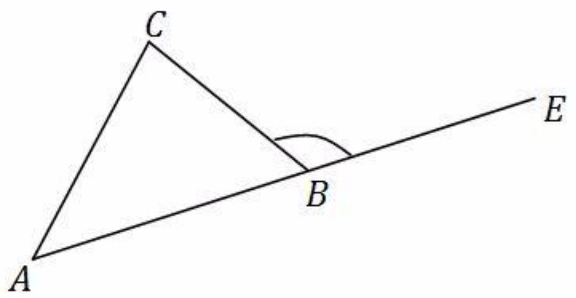
Interior angle – is an angle inside the triangle.The sum of interior angles of a triangle is

Example, consider the triangle below



$$\boxed{\angle ABC + \angle BCA + \angle CAB = 180^\circ}$$

Exterior angle - is an angle outside the triangle.Consider the triangle below



$\angle CBE$ – is an exterior angle of a triangle

From the triangle above

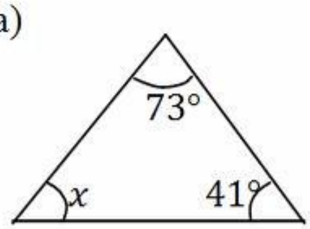
$$\angle CBE + \angle CBA = 180°$$

$\boxed{\angle CBE = 180° - \angle CBA}$ (angles on a straight line)

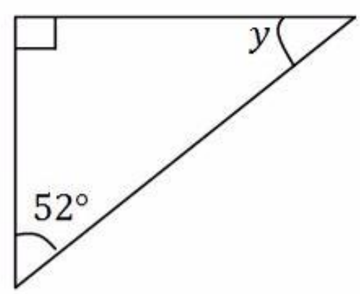
Example 2

Find the angles x and y in the diagrams below

(a)



(b)



Solution

(a) $x + 41° + 73° = 180°$

$$x + 114° = 180°$$
$$x = 180° - 114°$$
$$x = 66°$$

(b) $y + 52° + 90° = 180°$

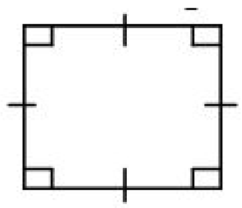
$$y + 142° = 180°$$
$$y = 180° - 142°$$
$$y = 38°$$

Different Quadrilaterals

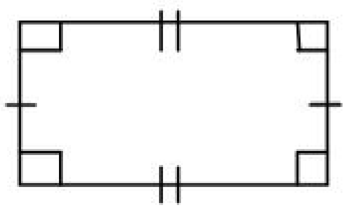
Construct different quadrilaterals

A quadrilateral – is a polygon with four sides.Examples of quadrilaterals are a square, a rectangle, a rhombus, a parallelogram, a kite and a trapezium

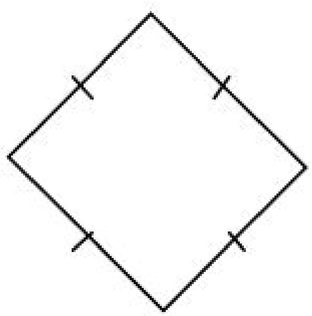
A square – has equal sides and all angles are 90º



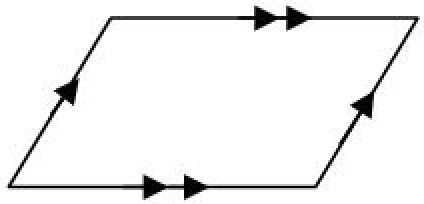
A rectangle – has two pairs of opposite sides equal and all angles are 90º



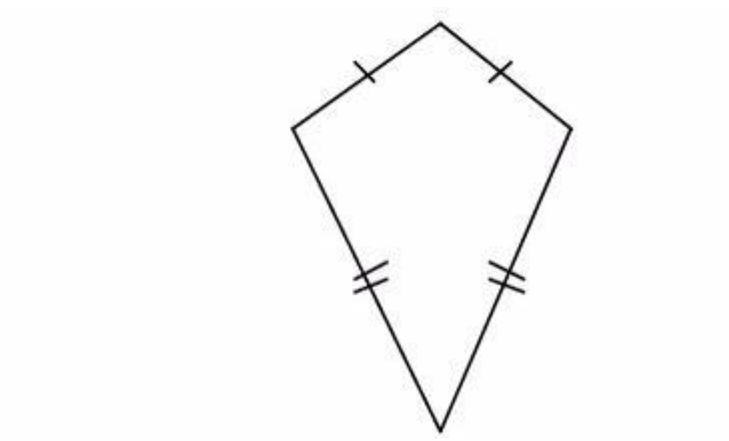
A rhombus – has all sides equal.Opposite angles are also equal



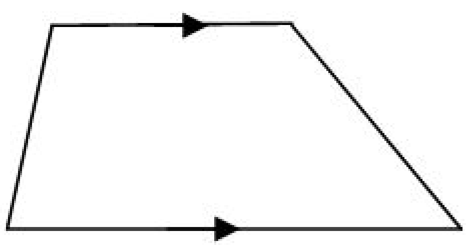
A parallelogram – has two pairs of opposite sides equal.Opposite angles are also equal



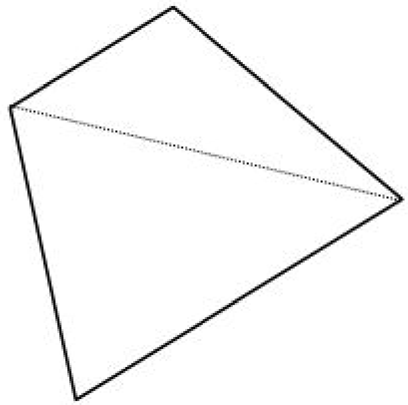
A kite – has two pairs of adjacent sides equal.One pair of opposite angles are also equal



A trapezium – has one pair of opposite sides pair



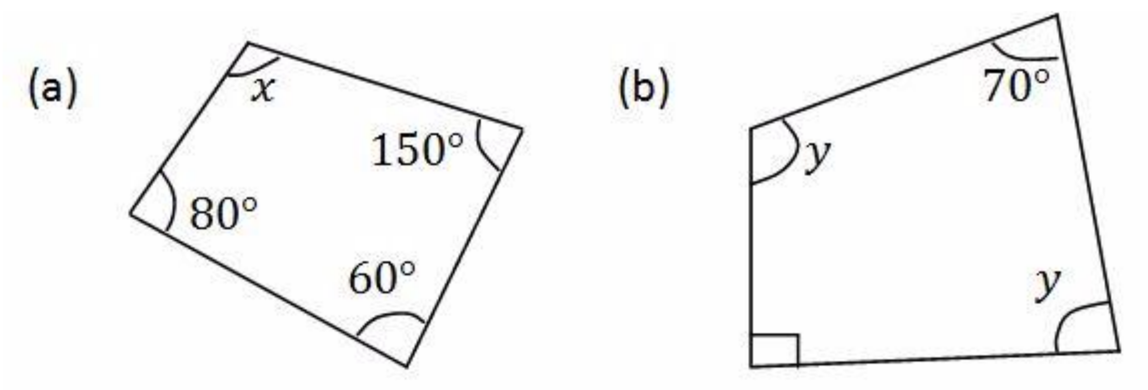
Any quadrilateral is made up of two triangles. Consider the below quadrilateral.



Sum of angles of quadrilateral = 2 ×180º = 360º

Example 3

Find the angles *x* and *y* in the diagrams below



Solution

(a) $x + 150° + 80° + 60° = 360°$

$$x + 290° = 360°$$
$$x = 360° - 290°$$
$$x = 70°$$

(b) $y + y + 70° + 90° = 360°$

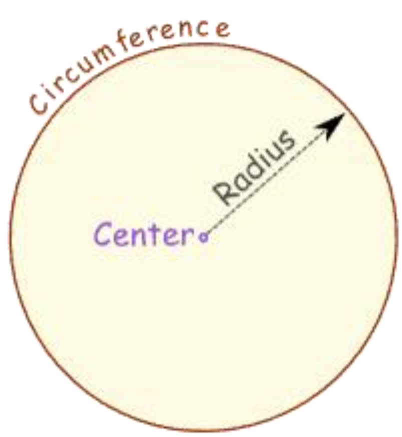
$$2y + 160° = 360°$$
$$2y = 360° - 160°$$
$$2y = 200°$$
$$y = \frac{200°}{2} = 100°$$
$$y = 100°$$

Circles

A Circle

Draw a circle

To make a circle: Draw a curve that is "radius" away from a central point.



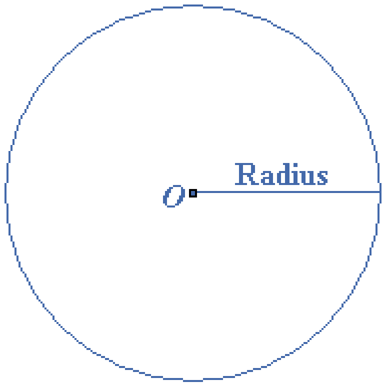
And so:All points are the same distance from the center.

You can draw it yourself:Put a pin in a board, put a loop of string around it, and insert a pencil into the loop. Keep the string stretched and draw the circle!

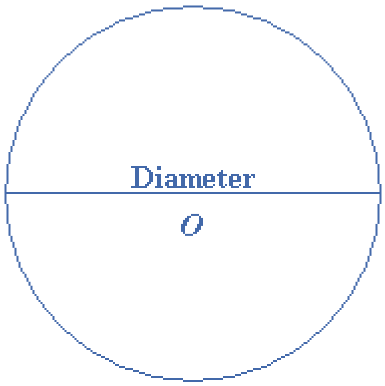
Different Parts of a Circle

Describe different parts of a circle

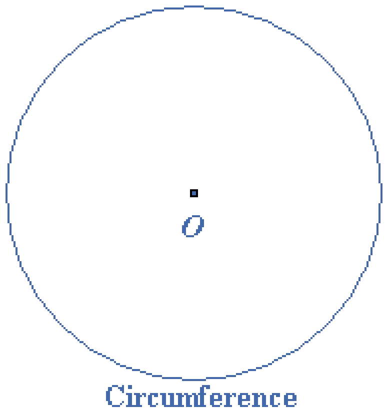
The **radius** of the circle is a straight line drawn from the center to the boundary line or the circumference. The plural of the word radius is **radii**.



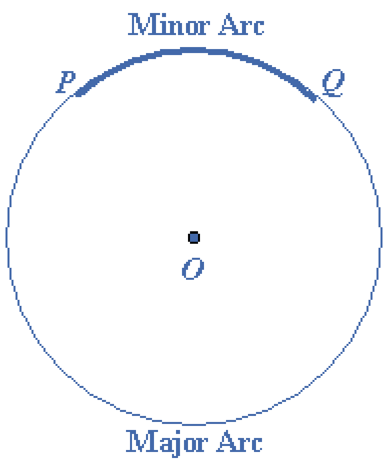
The **diameter** is the line crossing the circle and passing through the center. It is the twice of the length of the radius.



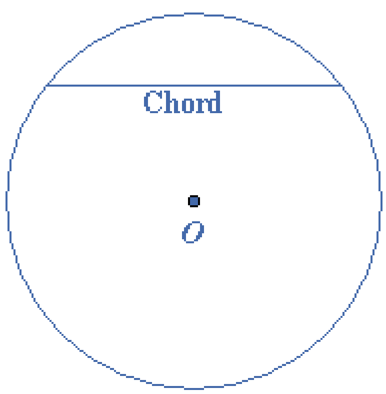
The **circumference** of a circle is the boundary line or the perimeter of the circle.



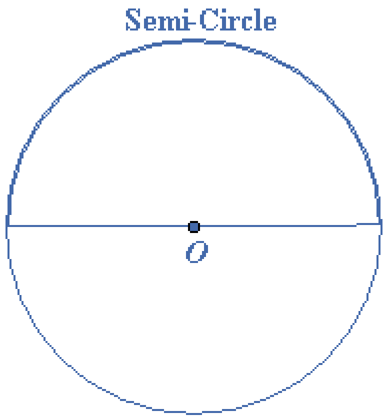
An **arc** is a part of the circumference between two points or a continuous piece of a circle. The shorter arc between and is called the **minor arc**. The longer arc between and is called the **major arc**.



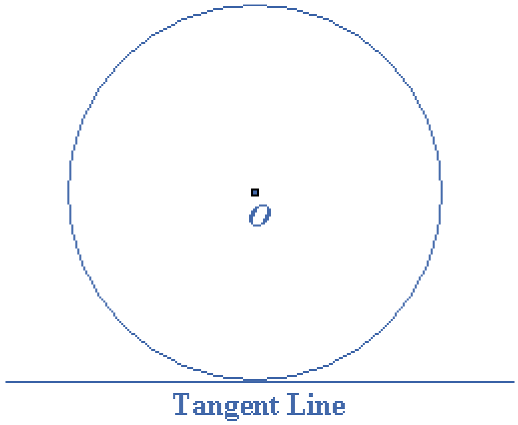
The **chord** is a straight line joining two points on the circumference points of a circle. The diameter is a special kind of the chord passing through the center.



A**semi-circle**is an arc which is half of thecircumference.



A **tangent** is a straight line which touches the circle. It does not cut the circumference. The point at which it touches, is called the **point of contact**.



ALGEBRA

An algebraic expression – is a collection of numbers, variables, operators and grouping symbols.Variables - are letters used to represent one or more numbers

Algebraic Operations

Symbols to form Algebraic Expressions

Use symbols to form algebraic expressions

The parts of an expression collected together are called **terms**

Example

- $x + 2x$ – are called like terms because they have the same variables
- $5x + 9y$ – are called unlike terms because they have different variables

An algebraic expression can be evaluated by replacing or substituting the numbers in the variables

Example 1

Evaluate the expressions below, given that $x = 2$ and $y = 3$

(a) $5x - y$
(b) $3(x + 2)$
(c) $\frac{1}{3}x + 2y$

Solution

(a) $5x - y = 5(2) - 3 = 10 - 3 = 7$
(b) $3(x + 2) = 3(2 + 2) = 3(4) = 12$
(c) $\frac{1}{3}x + 2y = \frac{1}{3}(2) + 2(3) = \frac{2}{3} + 6 = 6\frac{2}{3}$

Example 2

Evaluate the expressions below, given that m = 1 and n = - 2

(a) $\frac{3}{m+1}$

(b) $\frac{m}{1-n}$

(c) $m-n$

Solution

(a) $\frac{3}{m+1} = \frac{3}{1+1} = \frac{3}{2} = 1\frac{1}{2}$

(b) $\frac{m}{1-n} = \frac{1}{1- -2} = \frac{1}{1+2} = \frac{1}{3}$

(c) $m-n = 1- \ -2 = 1+2 = 3$

An expression can also be made from word problems by using letters and numbers

Example 3

A rectangle is 5 cm long and w cm wide. What is its area?

Solution

Let the area be A.

Then

A = length× widith

A = 5w cm^2

Simplifying Algebraic Expressions

Simplify algebraic expressions

Algebraic expressions can be simplified by addition, subtraction, multiplication and division

Addition and **subtraction** of algebraic expression is done by adding or subtracting the coefficients of the like terms or letters

Coefficient of the letter – is the number multiplying the letter

Multiplication and **division** of algebraic expression is done on the coefficients of both like and unlike terms or letters

Example 4

Simplify the expressions below

(a) $4a + 3b + 2a + b$

(b) $6n + 3m - 2n - 2m$

(c) $\frac{1}{2}s - t + 3s - \frac{1}{4}t$

(d) $5mn - 3nm$

Solution

(a) $4a + 3b + 2a + b$

Collect like terms together

$$\begin{aligned} 4a + 3b + 2a + b &= 4a + 2a + 3b + b \\ &= 6a + 4b \end{aligned}$$

(b) $6n + 3m - 2n - 2m$

Collect like terms together

$$\begin{aligned} 6n + 3m - 2n - 2m &= 6n - 2n + 3m - 2m \\ &= 4n + m \end{aligned}$$

(c) $\frac{1}{2}s - t + 3s - \frac{1}{4}t$

Collect like terms together

$$\begin{aligned} \frac{1}{2}s - t + 3s - \frac{1}{4}t &= \frac{1}{2}s + 3s - t - \frac{1}{4}t \\ &= 3\frac{1}{2}s - 1\frac{1}{4}t \end{aligned}$$

(d) $5mn - 3nm = 5mn - 3mn = 2mn$

Equations with One Unknown

An equation – is a statement that two expressions are equal

An Equation with One Unknown

Solve an equation with one unknown

An equation can have one variable *(unknown)* on one side or two variables on both sides.

When you shift a number or term from one side of equation to another, its sign changes

- If it is positive, it becomes negative
- If it is negative, it becomes positive

Example 5

Solve the following equations

(a) $x + 3 = 5$

(b) $x - 2\frac{1}{4} = 8$

(c) $3x + 4 = -3$

(d) $5x + 11 = 18$

(e) $5 - 3x = 24$

Solution

(a) $x + 3 = 5$

$$x = 5 - 3$$
$$x = 2$$

(b) $x - 2\frac{1}{4} = 8$

$$x = 8 + 2\frac{1}{4}$$
$$x = 8 + \frac{9}{4}$$
$$x = \frac{8}{1} + \frac{9}{4} = \frac{32 + 9}{4} = \frac{41}{4} = 10.25$$
$$x = 10.25$$

(c) $3x + 4 = -3$

$$3x = -3 - 4$$
$$3x = -7$$
$$x = -\frac{7}{3} = -2.33 \quad , \quad x = -2.33$$

(d) $5x + 11 = 18$

$$5x = 18 - 11$$
$$5x = 7$$
$$x = \frac{7}{5} = 1.4$$
$$x = 1.4$$

(e) $5 - 3x = 24$

$$-3x = 24 - 5$$
$$-3x = 19$$
$$x = \frac{19}{-3} = -\frac{19}{3} = -6.33$$
$$x = -6.33$$

An Equation from Word Problems

Form and solve an equation from word problems

Some word problems can be solved by using equations as shown in the below examples

Example 6

Naomi is 5 years young than Mariana. The total of their ages 33 years. How old is Mariana?

Solution

Let the age of Mariana be x

$$\text{Naomi} = x - 5$$

Then $$x + (x - 5) = 33$$

$$x + x - 5 = 33$$

$$2x = 33 + 5$$

$$2x = 38$$

$$x = \frac{38}{2} = 19$$

$$x = 19$$

Mariana is 19 years

Equations with Two Unknowns

Simultaneous Equations

Solve simultaneous equations

Simultaneous equations – are groups of equations containing multiple variables

Example 7

Examples of simultaneous equation

(ii) $\begin{cases} x + y = 1 \\ 2x - y = -7 \end{cases}$ (ii) $\begin{cases} 4t - 2s = 13 \\ -5t - s = 5 \end{cases}$ (iii) $\begin{cases} w - 9z = 40 \\ \frac{1}{3}w - \frac{1}{8}z = 34 \end{cases}$

A simultaneous equation can be solved by using two methods:

- Elimination method
- Substitution method

ELIMINATION METHOD

STEPS

- Choose a variable to eliminate*e.g x or y*
- Make sure that the letter to be eliminated has the same coefficient in both equations and if not, multiply the equations with appropriate numbers that will give the letter to be eliminated the same coefficient in both equations

(i) $\begin{cases} x - 3y = 8 \\ 2x + 3y = 4 \end{cases}$ eliminate y because it has the same coeffic

(ii) $\begin{cases} x - 5y = -1 \\ x + y = 2 \end{cases}$ eliminate x because it has the same coeffic

(iii) $\begin{cases} 5x + 4y = 2 \\ 3x + y = 3 \end{cases}$

- To eliminate x, multiply equation (i) by 3 and $(i$) by

$i.e$ $\left| \begin{array}{ll} 3 & 5x + 4y = 2 \\ 5 & 3x + y = 3 \end{array} \right.$

$$\rightarrow \begin{cases} 15x + 12y = 6 \\ 15x + 5y = 15 \end{cases}$$

- To eliminate y, multiply equation (i) by 1 and (ii) by 4

$i.e$

$$\left|\begin{array}{ll} 1 & 5x + 4y = 2 \\ 4 & 3x + y = 3 \end{array}\right.$$

$$\rightarrow \begin{cases} 5x + 4y = 2 \\ 12x + 4y = 12 \end{cases}$$

- If the signs of the letter to be eliminated are the same, subtract the equations
- If the signs of the letter to be eliminated are different, add the equations

Example 8

Solve the following simultaneous equations by elimination method

(a) $\begin{cases} 3x + y = 9 \\ 5x - y = 7 \end{cases}$

(b) $\begin{cases} 3x - 2y = 13 \\ 3x + 2y = 1 \end{cases}$

(c) $\begin{cases} 5r - g = 14 \\ 4r + 3g = 15 \end{cases}$

(d) $\begin{cases} 7x + 6y = 8 \\ 2x - 3y = 7 \end{cases}$

Solution

a. Eliminate y

$$+\left|\begin{array}{l}3x + y = 9 \ldots\ldots\ldots.(i)\\5x - y = 7 \ldots\ldots\ldots(ii)\end{array}\right.$$

$$8x = 16$$
$$x = \frac{16}{8} = 2$$
$$x = 2$$

To find y put x = 2 in either equation (i) or (ii)

From equation (i)

$$3x + y = 9$$
$$3(2) + y = 9$$
$$6 + y = 9$$
$$y = 9 - 6$$
$$y = 3$$

$\therefore \; x = 2 \;, \;\; y = 3$

(b)Eliminate x

$$-\left|\begin{array}{l}3x - 2y = 13 \ldots\ldots\ldots.(i)\\3x + 2y = 1 \ldots\ldots\ldots(ii)\end{array}\right.$$

$$-4y = 12$$
$$y = -12$$
$$y = -\frac{12}{4} = -3$$
$$y = -3$$

In order to find y, put x = 2 in either equation (i) or (ii)

From equation (ii)

$$3x + 2y = 1$$
$$3x + 2(-3) = 1$$
$$3x - 6 = 1$$
$$3x = 1 + 6$$
$$3x = 7$$
$$x = \frac{7}{3} = 2\frac{1}{3}$$
$$x = 2\frac{1}{3}$$

$\therefore\ x = 2\frac{1}{3}\ ,\ \ y = -3$

(c) Given

$$\begin{cases} 5r - g = 14 \ldots\ldots\ldots(i) \\ 4r + 3g = 15 \ldots\ldots\ldots(ii) \end{cases}$$

Eliminate g

$$\begin{array}{c|l} 3 & 5r - g = 14 \\ 1 & 4r + 3g = 15 \end{array}$$

$$+\left| \begin{array}{l} 15r - 3g = 42 \\ 4r + 3g = 15 \end{array} \right.$$

$$19\,r = 57$$

$$r = \frac{57}{19} = 3$$

$$r = 3$$

To find g put r = 3 in either equation (i) or (ii)

From equation *(i)*

$$5r - g = 14$$

$$5(3) - g = 14$$

$$15 - g = 14$$

$$-g = 14 - 15$$

$$-g = -1$$

$$g = 1$$

$\therefore\ r = 3\ ,\quad g = 1$

(d) Given

$$\begin{cases} 7x + 6y = 8 \dots\dots\dots (i) \\ 2x - 3y = 7 \dots\dots\dots (ii) \end{cases}$$

Eliminate x

$$\begin{array}{c|l} 2 & 7x + 6y = 8 \\ 7 & 2x - 3y = 7 \end{array}$$

$$- \left| \begin{array}{l} 14x + 12y = 16 \\ 14x - 21y = 49 \end{array} \right.$$

$$33y = -33$$

$$y = -\frac{33}{33} = -1$$

$$y = -1$$

To find x, put y = - 1 in either equation(i) or (ii)

From equation *(ii)*

$$2x - 3y = 7$$

$$2x - 3(-1) = 7$$

$$2x + 3 = 7$$

$$2x = 7 - 3$$

$$2x = 4$$

$$x = \frac{4}{2} = 2$$

$\therefore \ x = 2 \ , \quad y = -1$

BY SUBSTITUTION

STEPS

- Make the subject one letter in one of the two equation given e.g $\begin{cases} x - 3y = 8 \dots\dots\dots(\text{i}) \\ 2x + 3y = 4 \dots\dots..(\text{ii}) \end{cases}$

Make x the subject from equation (i)

$$x = 8 + 3y \dots\dots.(\text{iii})$$

- Substitute the letter in the remaining equation and proceed as in case of elimination

$$2x + 3y = 4$$
$$2(8 + 3y) + 3y = 4$$
$$16 + 6y + 3y = 4$$
$$16 + 9y = 4$$
$$9y = 4 - 16$$
$$9y = -12$$
$$y = -\frac{12}{9} = -\frac{4}{3}$$

From (iii)

$$x = 8 + 3y$$
$$x = 8 + 3\left(-\frac{4}{3}\right)$$
$$x = 8 - 4 = 4$$

$\therefore \quad x = 4 \text{ or } -\frac{4}{3}$

Example 9

Solve the following simultaneous equations by substitution method

(a) $\begin{cases} 2x + y = 17 \\ x + y = 4 \end{cases}$

Solution

(a) $\begin{cases} 2x + y = 17 \ldots\ldots\ldots \text{(i)} \\ x + y = 4 \ldots\ldots\ldots. \text{(ii)} \end{cases}$

From (ii)

$$x = 4 - y \ldots\ldots\ldots\ldots \text{(iii)}$$

Substitute (iii) into (i)

$$2x + y = 17$$
$$2(4 - y) + y = 17$$
$$8 - 2y + y = 17$$
$$8 - y = 17$$
$$-y = 17 - 8$$

$$-y = 9$$
$$y = -9$$

From (iii)

$$x = 4 - y$$
$$x = 4 - \ -9$$
$$x = 4 + 9 = 13$$

$\therefore \ x = 13, \quad y = -9$

Linear Simultaneous Equations from Practical Situations

Solve linear simultaneous equations from practical situations

Simultaneous equations can be used to solve problems in real life involving two variables

Example 10

If 3 Mathematics books and 4 English books weighs 24 kg and 5 Mathematics books and 2 English books weighs 20 kg, find the weight of one Mathematics book and one English book.

Solution

Let the weight of one Mathematics book = x and

Let the weight of one English book = y

Then

$$\begin{cases} 3x + 4y = 24 \dots\dots\dots (i) \\ 5x + 2y = 20 \dots\dots\dots (ii) \end{cases}$$

Eliminate y

$$\begin{array}{r|l} 2 & 3x + 4y = 24 \\ 4 & 5x + 2y = 20 \end{array}$$

$$- \left| \begin{array}{l} 6x + 8y = 48 \\ 20x + 8y = 80 \end{array} \right.$$

$$-14x = -32$$

$$x = \frac{-32}{-14} = 2.29$$

$$x = 2.29$$

To find y, put x = 2.29 in either equation (i) or (ii)

From equation*(i).*

$$3x + 4y = 24$$
$$3(2.29) + 4y = 24$$
$$6.87 + 4y = 14$$
$$4y = 24 - 6.87$$
$$4y = 17.13$$
$$y = \frac{17.13}{4} = 4.28$$

$\therefore \; x = 2.29 \;, \quad y = 4.28$

Inequalities

An inequality – is a mathematical statement containing two expressions which are not equal. One expression may be less or greater than the other.The expressions are connected by the inequality symbols$<,>,\leq$ or$\geq$.Where$<$ = less than,$>$ = greater than,$\leq$ = less or equal and $\geq$ = greater or equal.

Linear Inequalities with One Unknown

Solve linear inequalities in one unknown

An inequality can be solved by collecting like terms on one side.Addition and subtraction of the terms in the inequality does not change the direction of the inequality.Multiplication and division of the sides of the inequality by a positive number does not change the direction of the inequality.But multiplication and division of the sides of the inequality by a negative number changes the direction of the inequality

Example 11

Solve the following inequalities

(a) $x+8>15$ (b) $7-2x<11$ (c) $\frac{1}{5}x-3\geq -2+x$ (d) $3x -$

(e) $\frac{2x+8}{-3}\geq 20$ (f) $\frac{1}{2}x-4\leq 3-\frac{2}{3}x$

Solution

(a) $x+8>15$

$$x>15-8$$

$$x>7$$

(b) $7-2x<11$

$$-2x<11-7$$

$$-2x<4$$

$$x>-2$$

(c) $\frac{1}{5}x-3\geq -2+x$

Collect like terms

$$\frac{1}{5}x-x\geq -2+3$$

$$-\frac{4}{5}x \geq -1$$

$$x\leq \frac{5}{4}$$

(d) $3x - 1 < 2x - 5$

Collect like terms

$$3x - 1 < 2x - 5$$
$$3x - 2x < -5 + 1$$
$$x < -4$$

(e) $\frac{2x+8}{-3} \geq 20$

Multiply by -3 both sides

$$2x + 8 \leq -60$$

$$2x \leq -60 - 8$$

$$2x \leq -68$$

$$x \leq -34$$

(f) $\frac{1}{2}x - 4 \leq 3 - \frac{2}{3}x$

Collect like terms

$$\frac{1}{2}x - 4 \leq 3 - \frac{2}{3}x$$
$$\frac{1}{2}x + \frac{2}{3}x \ \leq 3 + 4$$
$$\frac{3x + 4x}{6} \ \leq 3$$
$$\frac{7}{6}x \ \leq \ 3$$
$$x \ \leq \ 3\left(\frac{6}{7}\right)$$
$$x \ \leq \frac{18}{7}$$

Linear Inequalities from Practical Situations

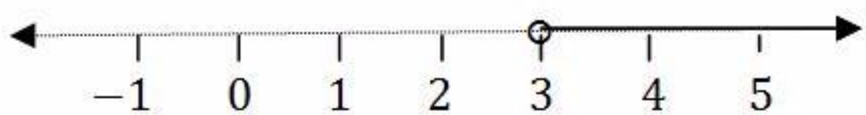
Form linear inequalities from practical situations

To represent an inequality on a number line, the following are important to be considered:

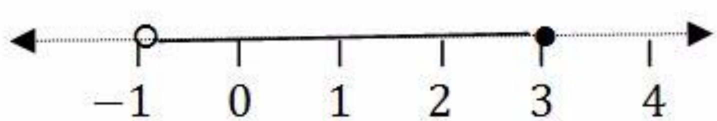
- The endpoint which is not included is marked with an empty circle
- The endpoint which is included is marked with a solid circle

Example 12

(a) $x > 3$



(b) $-1 < x \leq 3$



Compound statement – is a statement made up of two or more inequalities

Example 13

Solve the following compound inequalities and represent the answer on the number line

(a) $10 \leq 2x - 3 < 14$

(b) $7 \leq 3 - 2x < 15$

Solution

(a) $10 \leq 2x - 3 < 14$

Express in the form of $a < x < b$

Add 3 on each part of the inequality

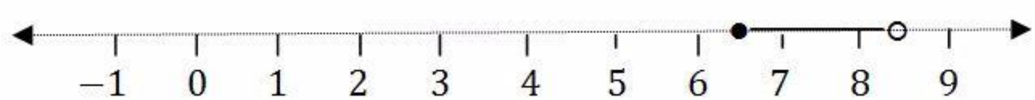
$$10 + 3 \leq 2x - 3 + 3 < 14 + 3$$

$$13 \leq 2x < 17$$

Divide by 2 each part of the inequality

$$\frac{13}{2} \leq x < \frac{17}{2}$$

$$6\frac{1}{2} \leq x < 8\frac{1}{2}$$



(b) $7 \leq 3 - 2x < 15$

Subtract 3 on each part of the inequality

$$7 - 3 \leq 3 - 3 - 2x < 15 - 3$$

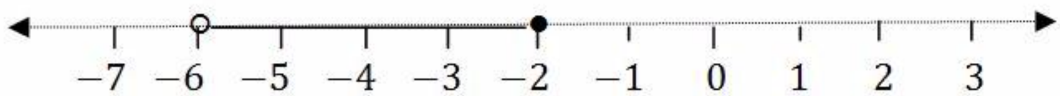
$$4 \leq -2x < 12$$

Divide by -2 each part of the inequality

$$\frac{4}{-2} \geq x > \frac{12}{-2}$$

$$-2 \geq x > -6$$

$$-6 < x \leq -2$$



NUMBERS (II)

Rational Numbers

A Rational Number

Define a rational number

A **Rational Number** is a real number that can be written as a simple fraction (i.e. as a **ratio**). Most numbers we use in everyday life are Rational Numbers.

Number	**As a Fraction**	**Rational?**
5	5/1	Yes
1.75	7/4	Yes
.001	1/1000	Yes
-0.1	-1/10	Yes
0.111...	1/9	Yes
√2(square root of 2)	?	**NO !**

The square root of 2 cannot be written as a simple fraction! And there are many more such numbers, and because they are**not rational**they are calledIrrational.

The Basic Operations on Rational Numbers

Perform the basic operations on rational numbers

Addition of Rational Numbers:

To add two or morerational numbers, the denominator of all the rational numbers should be the same. If the denominators of all rational numbers are same, then you can simply add all the numerators and the denominator value will the same. If all the denominator values are not the same, then you have to make the denominator value as same, by multiplying the numerator and denominator value by a common factor.

Example 1

$1/3+4/3=5/3$

$1/3 + 1/5 = 5/15 + 3/15 = 8/15$

Subtraction of Rational Numbers:

To subtract two or more rational numbers, the denominator of all the rational numbers should be the same. If the denominators of all rational numbers are same, then you can simply subtract the numerators and the denominator value will the same. If all the denominator values are not the same, then you have to make the denominator value as same by multiplying the numerator and denominator value by a common factor.

Example 2

$4/3 - 2/3 = 2/3$

$1/3-1/5=5/15-3/15=2/15$

Multiplication of Rational Numbers:

Multiplication of rational numbers is very easy. You should simply multiply all the numerators and it will be the resulting numerator and multiply all the denominators and it will be the resulting denominator.

Example 3

$4/3 x 2/3=8/9$

Division of Rational Numbers:

Division of rational numbers requires multiplication of rational numbers. If you are dividing two rational numbers, then take the reciprocal of the second rational number and multiply it with the first rational number.

Example 4

4⁄3÷2⁄5=4⁄3x5⁄2=20⁄6=10⁄3

Irrational Numbers

Irrational Numbers

Define irrational numbers

An irrational number is areal numberthat cannot be reduced to any ratio between anintegerpand anatural number*q*. The union of the set of irrational numbers and the set of rational numbers forms the set of real numbers. In mathematical expressions, unknown or unspecified irrationals are usually represented by*u*through*z*. Irrational numbers are primarily of interest to theoreticians. Abstract mathematics has potentially far-reaching applications in communications and computer science, especially in data encryption and security.

Examples of irrational numbers are √2 (the square root of 2), the cube root of 3, the circular ratio , and the naturallogarithmbase*e*. The quantities√2 and the cube root of 3 are examples of algebraic numbers. Pi and *e* are examples of special irrationals known as atranscendental numbers. The decimal expansion of an irrational number is always nonterminating (it never ends) and non repeating (the digits display no repetitive pattern)

Real Numbers

Real Numbers

Define real numbers

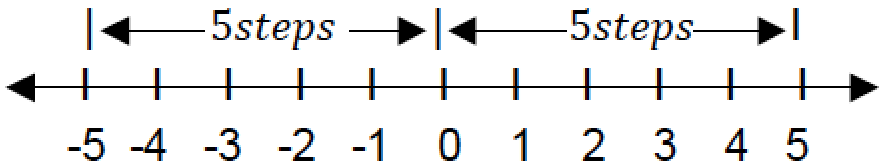
he type of number we normally use, such as 1, 15.82, −0.1, 3/4, etc.Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers.

They are called "Real Numbers" because they are not Imaginary Numbers.

Absolute Value of Real Numbers

Find absolute value of real numbers

The absolute value of a number is the magnitude of the number without regard to its sign. For example, the absolute value of x or x written as . The sign before x is ignored. This is because the distance represented is the same whether positive or negative. For example, a student walking 5 steps forward or 5 steps backwards will be considered to have moved the same distance from where she originally was, regardless of the direction.



The 5 steps forward (+5) and 5 steps backward (-5) have an absolute value of 5

Thus $|x| = x$ when x is positive ($x \geq 0$), but $|x| = -x$ when x is negative ($x \leq 0$).

For example, $|3| = 3$ since 3 is positive ($3 \geq 0$) And $-3 = (-3) = 3$ since -3 is negative ($3 \leq 0$)

Related Practical Problems

Solve related practical problems

Example 5

Solve for x if $|x| = 5$

Solution

For any number x, $|x| = 5$, there are two possible values. Either $x, = +5$ or $x = 5$

Example 6

Solve for x, given that $|x + 2| = 4$

Solution

Either $+(x+2) = 4$ or $-(x+2) = 4$

$\rightarrow x = 4 - 2$ $\quad$ $-x - 2 = 4$

$= 2$ $\quad$ $= -6$

Therefore, $x = 2$ or -6

RATIO, PROFIT AND LOSS

Ratio

A ratio – is a way of comparing quantities measured in the same units

Examples of ratios

1. A class has 45 girls and 40 boys. The ratio of number of boys to the number of girls = 40: 45

2. A football ground 100 m long and 50 m wide. The ratio of length to the width = 100: 50

NOTE: Ratios can be simplified like fractions

1. 40: 45 = 8: 9

2. 100: 50 = 2: 1

A Ratio in its Simplest Form

Express a ratio in its simplest form

Example 1

Simplify the following ratios, giving answers as whole numbers

(a) $17: 34$

(b) $2.4 : 1.4$

(c) $5.6 : 2.4$

(d) $\frac{2}{3} : \frac{4}{9}$

(e) $\frac{3}{8} : \frac{9}{16}$

Solution

(a) Divide by 17 each number $17:34 = 1:2$

(b) Multiply by 10 each number $2.4 : 1.4 = 24 : 14$

Divide by 2 $24 : 14 = 12:7$

(c) Multiply by 10 each number $5.6 : 2.4 = 56 : 24$

Divide by 8 $56 : 24 = 7:3$

(d) Multiply by 9 each fraction $\frac{2}{3} : \frac{4}{9} = 6 : 4$

Divide by 2 $6 : 4 = 3:2$

(e) Multiply by 16 each fraction $\frac{3}{8} : \frac{9}{16} = 6 : 9$

Divide by 3 $6 : 9 = 2:3$

A Given Quantity into Proportional Parts

Divide a given quantity into proportional parts

Example 2

Express the following ratios in the form of

(a) $0.8 : 1.6$

(b) $55 : 11$

(c) $500 : 250$

(d) $\frac{2}{3} : \frac{1}{6}$

(e) $\frac{3}{4} : \frac{5}{12}$

Solution

(a) Divide by 1.6 each number $0.8 : 1.6 = \frac{0.8}{1.6} : \frac{1.6}{1.6} = 0.5 : 1$

(b) Divide by 11 each number $55 : 11 = \frac{55}{11} : \frac{11}{11} = 5 : 1$

(c) Divide by 250 each number $500 : 250 = \frac{500}{250} : \frac{250}{250} = 2 : 1$

(d) Multiply by 6 each fraction $\frac{2}{3} : \frac{1}{6} = 4 : 1$

(e) Multiply by 12 each fraction $\frac{3}{4} : \frac{5}{12} = 9 : 5$

Divide by 5 $9 : 5 = \frac{9}{5} : \frac{5}{5} = 1.8 : 1$

To increase or decrease a certain quantity in a given ratio, multiply the quantity with that ratio

Example 3

a. Increase 6 m in the ratio 4 : 3

b. Decrease 800 /− in the ratio 4 : 5

Solution

(a) $6\text{m} \times \frac{4}{3} = 8\,m$

(b) $800/- \times \frac{4}{5} = 640/-$

Profits and Loss

Profit or Loss

Find profit or loss

If you buy something and then sell it at a higher price, then you have a profit which is given by: Profit = selling price − buying price

If you buy something and then sell it at a lower price, then you have a loss which is given by: Loss = buying price − selling price

The profit or loss can also be expressed as a percentage of buying price as follows:

$$\boxed{\text{Percentage profit} = \frac{\text{profit}}{\text{buying price}} \text{ x } 100\%}$$

And

$$\boxed{\text{Percentage loss} = \frac{\text{loss}}{\text{buying price}} \text{ x } 100\%}$$

Percentage Profit and Percentage Loss

Calculate percentage profit and percentage profit and percentage loss

Example 4

Mr. Richard bought a car for 3, 000, 000/− and sold for 3, 500, 000/−. What is the profit and percentage profit obtained?

Solution

Profit= selling price − buying price = 3,500,000-3,000,000=500,000

Therefore the profit obtained is 500,000/-

$$\text{Percentage profit} = \frac{\text{profit}}{\text{buying price}} \text{ x } 100\%$$

But buying price $= 3,000,000/-$ and

Profit $= 500,000/-$

$$\therefore \text{Percentage profit} = \frac{500,000}{3,000,000} \text{ x } 100\% = \frac{1}{6} \text{ x } 100\% = \frac{100}{6}\% = 16.67\%$$

Example 5

Eradia bought a laptop for

Solution

$$\text{Percentage loss} = \frac{\text{loss}}{\text{buying price}} \ \text{x} \ \ 100\%$$

But buying price = 780, 000/− and loss = buying price − selling price = 780, 000 − 720, 000 = 60, 000/−

$$\therefore \text{Percentage loss} = \frac{60,000}{780,000} \ \text{x} \ \ 100\% = \frac{1}{13} \ \text{x} \ 100\% = \frac{100}{13}\% = 7.69\%$$

Simple Interest

Simple Interest

Calculate simple interest

The amount of money charged when a person borrows money e. g from a bank is called interest (I)

The amount of money borrowed is called principle (P)

To calculate interest, we use interest rate (R) given as a percentage and is usually taken per year or per annum (p.a)

$$\boxed{I = \frac{PRT}{100}}$$

Example 6

Calculate the simple interest charged on the following

a. 850, 000/− at 15% per annum for 9 months

b. 200, 000/− at 8% per annum for 2 years

Solution

(a) $P = 850,000/-, \quad R = 15\% \quad T = 9 \text{ months}$

Change time from months to years

$$1\ year = 12\ months$$

$$?\quad = 9\ months$$

$$= \frac{1\ year\ x\ 9\ months}{12\ months} = \frac{9}{12}\ years$$

$$T = \frac{9}{12}\ years$$

$$I = \frac{PRT}{100} = \frac{850,000\ \text{x}\ 15\ \text{x}\ \frac{9}{12}}{100} = \frac{850,000\ \text{x}\ 15\ \text{x}\ 0.75}{100} = 95\ 625/-$$

(b) $P = 200,000/-, \quad R = 8\% \quad T = 2 \text{ years}$

$$I = \frac{PRT}{100} = \frac{200,000\ \text{x}\ 8\ \text{x}\ 2}{100} = 32\ 00/-$$

Real Life Problems Related to Simple Interest

Solve real life problems related to simple interest

Example 7

Mrs. Mihambo deposited money in CRDB bank for 3 years and 4 months. A t the end of this time she earned a simple interest of 87, 750/− at 4.5% per annum. How much had she deposited in the bank?

Solution

Given I = 87, 750/− R = 4.5% % T = 3 years and 4 months

Change months to years

$$1\ year = 12\ months$$
$$? \quad = 4\ months$$
$$= \frac{1\ year\ x\ 4\ months}{12\ months} = \frac{4}{12}\ years = 0.3\ years$$

$$T = (3 + 0.3) years = 3.3 years$$

$$I = \frac{PRT}{100} \quad \rightarrow 100I = PRT$$

$$P = \frac{100I}{RT} = \frac{100\ x\ 87,750}{4.5\ \ x\ \ 3.3} = \frac{8775000}{14.85} = 590\ 909/-$$

$\therefore$ She deposited 590 909/−

COORDINATE GEOMETRY

Coordinates of a Point

The Coordinates of a Point

Read the coordinates of a point

Coordinates of a points – are the values of x and y enclosed by the bracket which are used to describe the position of a point in the plane

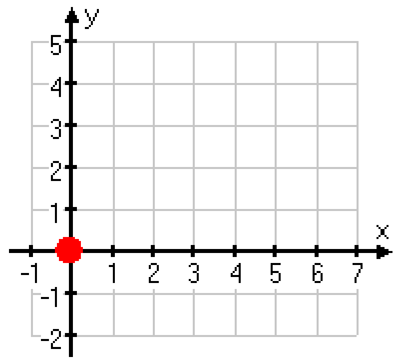
The plane used is called $xy -$ plane and it has two axis; horizontal axis known as $x -$ axis and; vertical axis known as $y -$ axis

A Point Given its Coordinates

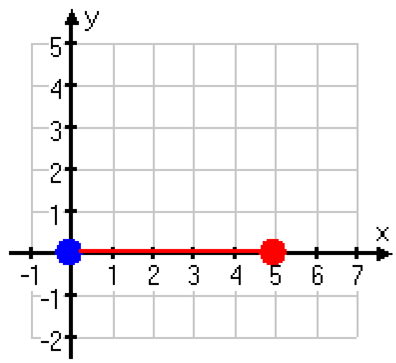
Plot a point given its coordinates

Suppose you were told to locate (5, 2) on the plane. Where would you look? To understand the meaning of (5, 2), you have to know the following rule: The*x*-coordinate (*always*comes first. The first number (the first coordinate) is*always*on the horizontal axis.

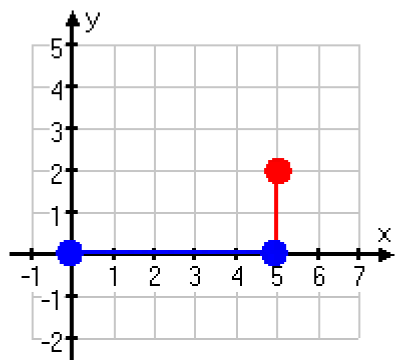
So, for the point $(5, 2)$, you would start at the "origin", the spot where the axes cross:



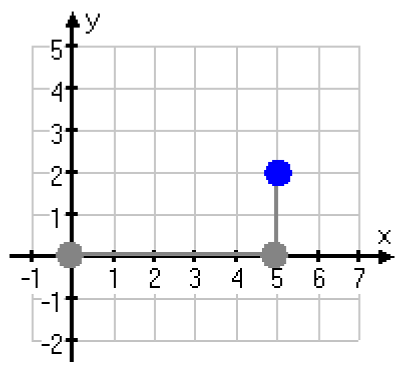
...then count over to "five" on the x-axis:



...then count up to "two", moving parallel to the y-axis:



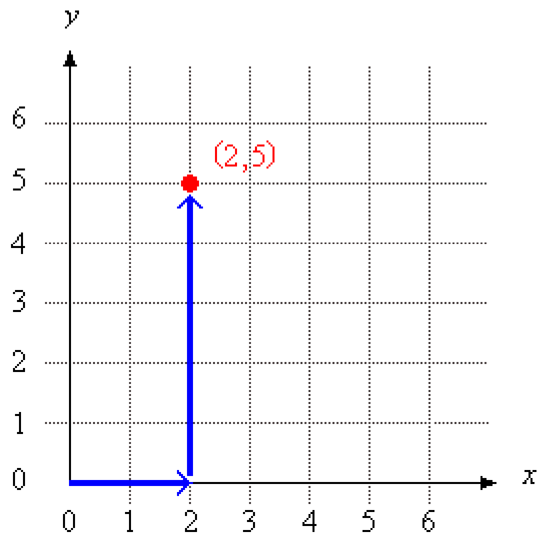
...and then draw in the dot:



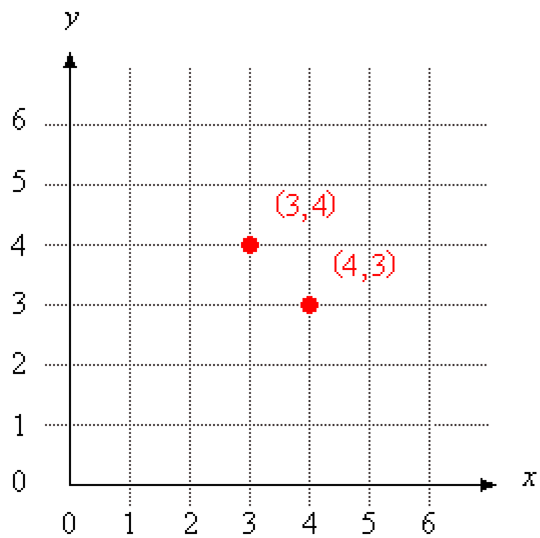
A Point on the Coordinates

Locate a point on the coordinates

The location of (2,5) is shown on the coordinate grid below. The *x*-coordinate is 2. The *y*-coordinate is 5. To locate (2,5), move 2 units to the right on the *x*-axis and 5 units up on the *y*-axis.



The order in which you write *x*- and *y*-coordinates in an ordered pair is very important. Th e*x*-coordinate always comes first, followed by the *y*-coordinate. As you can see in the coordinate grid below, the ordered pairs (3,4) and (4,3) refer to two different points!



Gradient (Slope) of a Line

The Gradient of a Line Given Two Points

Calculate the gradient of a line given two points

Gradient or slope of a line – is defined as the measure of steepness of the line. When using coordinates, gradient is defined as change in y to the change in x.

$$\boxed{\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}}$$

Consider two points A (x_1, $y_{1)}$and (B x_2, $y_{2)}$, the slope between the two points is given by:

$$\boxed{\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}}$$

OR

$$\boxed{\text{Gradient} = \frac{y_1 - y_2}{x_1 - x_2}}$$

Example 1

Find the gradient of the lines joining:

a. (5, 1) and (2,−2)

b. (4,−2) and (−1, 0)

c. (−2,−3) and (−4,−7)

Solution

(a) $(5,1)$ and $(2,-2)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{2-5} = \frac{-3}{-3} = 1$$

(b) $(4,-2)$ and $(-1,0)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - -2}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$$

(c) $(-2,-3)$ and $(-4,-7)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - \ -3}{-4 - -2} = \frac{-7+3}{-4+2} = \frac{-4}{-2} = 2$$

Example 2

a. The line joining (2,−3) and (k, 5) has gradient −2. Find k

b. Find the value of m if the line joining the points (−5,−3) and (6,m) has a slope of½

Solution

(a) Given $(2, -3)$ and $(k, 5)$

gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$$-2 = \frac{5 - -3}{k - 2}$$
$$-2(k - 2) = 5 + 3$$
$$-2k + 4 = 8$$
$$-2k = 8 - 4$$
$$-2k = 4$$
$$k = \frac{4}{-2} = -2$$

$\therefore$ The value of k is -2

(b) Given $(-5, -3)$ and $(6, m)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{1}{2} = \frac{m - -3}{6 - -5}$$

$$\frac{1}{2} = \frac{m + 3}{6 + 5}$$

$$\frac{1}{2} = \frac{m + 3}{11}$$

$$2(m + 3) = 11$$
$$2m + 6 = 11$$
$$2m = 11 - 6$$
$$2m = 5$$
$$m = \frac{5}{2}$$

The value of k is $\frac{5}{2}$

The Equations of a Line Given the Coordinates of Two Points on a Line

Find the equations of a line given the coordinates of two points on a line

The equation of a straight line can be determined if one of the following is given:-

- The gradient and the y − intercept (at x = 0) or x − intercept (at y=0)
- The gradient and a point on the line
- Since only one point is given, then

$$\boxed{\text{gradient} = \frac{y - y_1}{x - x_1}}$$

- Two points on the line

Example 3

Find the equation of the line with the following

a. Gradient 2 and y − intercept −4

b. Gradient −2⁄3and passing through the point (2, 4)

c. Passing through the points (3, 4) and (4, 5)

Solution

(a) Given $m = 2$ and $c = -4$

$$y = mx + c$$
$$y = 2x - 4$$

(b) Recall

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{2}{3} = \frac{y - 4}{x - 2}$$

$$-2(x - 2) = 3(y - 4)$$

$$-2x + 4 = 3y - 12$$

$$-2x + 4 - 3y + 12 = 0$$

$$-2x - 3y + 16 = 0$$

Divide by the negative sign, (−), throughout the equation

∴The equation of the line is $2x + 3y - 16 = 0$

(c) Recall

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-4}{4-3} = \frac{1}{1} = 1$$

Then

$$\text{gradient} = \frac{y - y_1}{x - x_1}$$

$$1 = \frac{y-4}{x-3}$$

$$x - 3 = y - 4$$

$$x - 3 - y + 4 = 0$$

$$x - y + 1 = 0$$

∴The equation of the line is $x - y + 1 = 0$

The equation of a line can be expressed in two forms

a. $ax + by + c = 0$ and

b. $y = mx + c$

Consider the equation of the form $y = mx + c$

m = Gradient of the line

Example 4

Find the gradient of the following lines

a. $2y = 5x + 1$

b. $2x + 3y = 5$

c. $x + y = 3$

Solution

(a) Express in the form of $y = mx + c$

Divide by both sides

$$y = \frac{5x + 1}{2} = \frac{5}{2}x + \frac{1}{2}$$
$$y = \frac{5}{2}x + \frac{1}{2}$$

$\therefore$ Gradient $= \frac{5}{2}$

(b) Express in the form of $y = mx + c$

Divide by both sides

$$2x + 3y = 5$$
$$3y = 5 - 2x$$
$$3y = -2x + 5$$
$$y = \frac{-2x + 5}{3} = -\frac{2}{3}x + \frac{5}{3}$$
$$y = -\frac{2}{3}x + \frac{5}{3}$$

$\therefore$ Gradient $= \frac{5}{2}$

(c) $x + y = 3$

Express in the form of $y = mx + c$

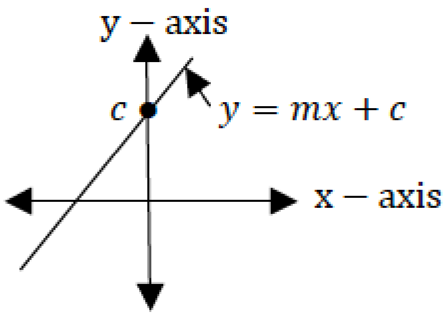
$$y = 3 - x$$
$$y = -x + 3$$

$\therefore$ Gradient $= -1$

Intercepts

The line of the form $y = mx + c$, crosses the $y - axis$ when $x = 0$ and also crosses $x - axis$ when $y = 0$

See the figure below



Therefore

a. to get x − intercept, let y = 0 and

b. to get y − intercept, let x = 0

From the line, $y = mx + c$

y − intercept, let $x = 0$

$y = m\ 0 + c = 0 + c = c$

y − intercept = c

Therefore, in the equation of the form $y = mx + c$, m is the gradient and c is the y − intercept

Example 5

Find the y − intercepts of the following lines

(a) $y = 3x + 5$

(b) $y = -\frac{1}{2}x + \frac{2}{3}$

(c) $3y = 2x + 1$

Solution

(a) $y = 3x + 5$

Compare with $y = mx + c$

$$y - \text{intercept} = c = 5$$

$\therefore y - \text{intercept is } 5$

(b) $y = -\frac{1}{2}x + \frac{2}{3}$

$$y - \text{intercept} = \frac{2}{3}$$

(c) $3y = 2x + 1$

Express in the form of $y = mx + c$

Divide by 3 both sides

$$y = \frac{2x+1}{3} = \frac{2}{3}x + \frac{1}{3}$$

$$y = \frac{2}{3}x + \frac{1}{3}$$

$$y - \text{intercept} = \frac{1}{3}$$

Graphs of Linear Equations

The Table of Value

Form the table of value

The graph of a straight line can be drawn by using two methods:

a. By using intercepts

b. By using the table of values

Example 6

Sketch the graph of $y = 2x - 1$

Solution

By using intercepts

$y -$ intercept, let $x = 0$

$$y = 2(0) - 1$$
$$y = 0 - 1$$
$$y = -1$$

$x -$ intercept, let $y = 0$

$$0 = 2x - 1$$
$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

The coordinates are $\left(\frac{1}{2}, 0\right)$ and (0, -1)

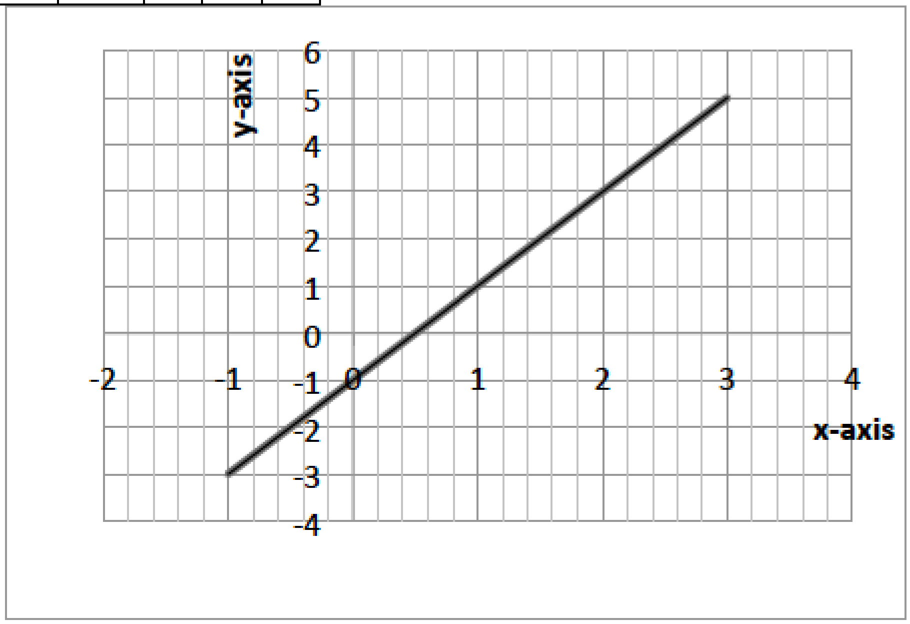
Then show the straight line through the point $\left(\frac{1}{2}, 0\right)$ and (0, -1) on the xy −plane.

The Graph of a Linear Equation

Draw the graph of a linear equation

By using the table of values

x	-1	0	1	2	3
y	-3	-1	1	3	5



Simultaneous Equations

Linear Simultaneous Equations Graphically

Solve linear simultaneous equations graphically

Use the intercepts to plot the straight lines of the simultaneous equations. The point where the two lines cross each other is the solution to the simultaneous equations

Example 7

Solve the following simultaneous equations by graphical method

$$\begin{cases} x + y = 4 \\ 2x - y = 2 \end{cases}$$

Solution

Consider: $x + y = 4$

If $x = 0$, $0 + y = 4$ $y = 4$

If $y = 0$, $x + 0 = 4$ $x = 4$

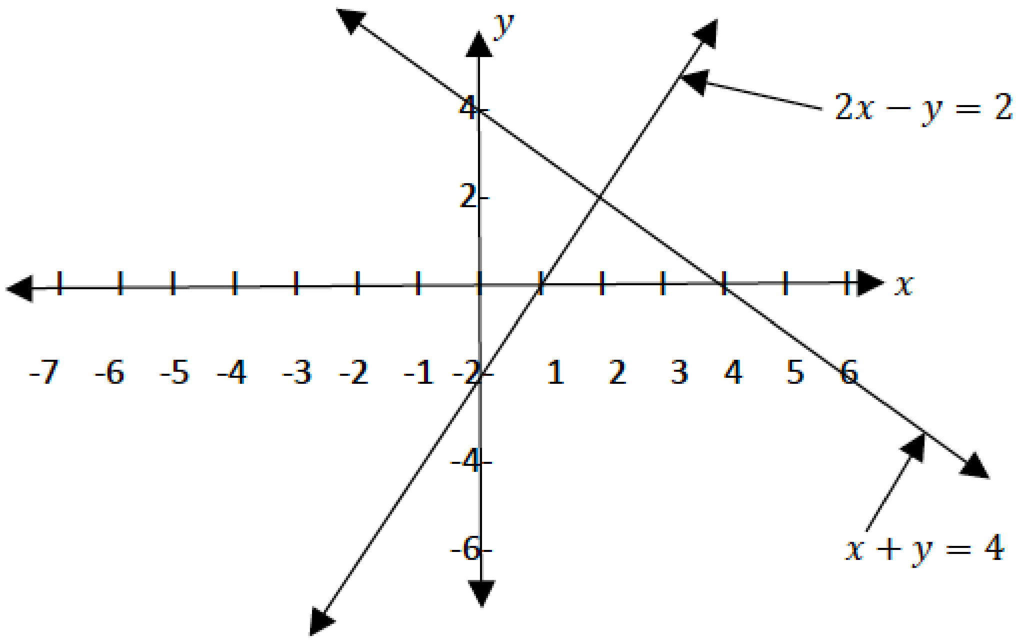
Draw a straight line through the points 0, 4 and 4, 0 on the xy − plane

Consider: $2x - y = 2$

If $x = 0$, $0 - y = 2$ $y = -2$

If $y = 0$, $2x - 0 = 2$ $x = 1$

Draw a straight line through the points $(0,-2)$ and $(1, 0)$ on the xy − plane



From the graph above the two lines meet at the point 2, 2 , therefore $x = 2$ and $y = 2$

PERIMETERS AND AREAS

Perimeters of Triangles and Quadrilaterals

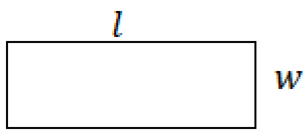
The Perimeters of Triangles and Quadrilaterals

Find the perimeters of triangles and quadrilaterals

Perimeter – is defined as the total length of a closed shape. It is obtained by adding the lengths of the sides inclosing the shape. Perimeter can be measured in m , cm ,dm ,m,km e. t. c

Examples

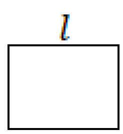
(i) A rectangle of length l and width w



$$\text{Perimeter, P} = l + l + \text{w} + \text{w} = 2\text{l} + 2\text{w} = 2(l + \text{w})$$

$$\boxed{\text{P} = 2(l + w)}$$

(ii) A square of side l



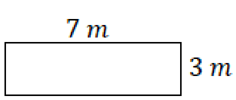
$$\text{Perimeter, P} = l + l + l + l = 4l$$

$$\boxed{\text{P} = 4l}$$

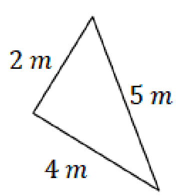
Example 1

Find the perimeters of the following shapes

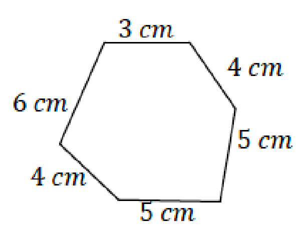
(a)



(b)



(c)



Solution

a. Perimeter = $7m + 7m + 3m + 3m = 20\ m$

b. Perimeter = $2m + 4m + 5m = 11\ m$

c. Perimeter = $3cm + 6cm + 4cm + 5cm + 5\ cm + 4cm = 27\ cm$

Circumference of a Circle

The Value of Pi (Π)

Estimate the value of Pi (Π)

The number π is a mathematical constant, the ratio of a circle's circumference to its diameter, commonly approximated as**3.14159**. It has been represented by the Greek letter "π" since the mid 18th century, though it is also sometimes spelled out as "pi" (/paɪ/).

The perimeter of a circle is the length of its circumference $i.e\ perimeter = circumference$. Experiments show that the ratio of the circumference to the diameter is the same for all circles

$$i.e \qquad \frac{\text{circumference}}{\text{diameter}} = \text{constant number called}, \pi$$

$$\frac{C}{d} = \pi$$

$$\boxed{C = \pi d}$$

Where c = circumference of a circle, d = diameter of the circle

But $d = 2r$, then

$$\boxed{C = 2\pi r}$$

Where r = radius of the circle

The Circumference of a Circle

Calculate the circumference of a circle

Example 2

Find the circumferences of the circles with the following measurements. Take π = 3.14

a. diameter 9 cm

b. radius 3½m

c. diameter 4.5 dm

d. radius 8 km

Solution

(a) $C = \pi d = 3.14 \ x \ 9 = 28.26 \, cm$

(b) $C = 2\pi r = 2 \ x \ 3.14 \ x \ 3\frac{1}{2} = 6.28 \ x \ \frac{7}{2} = 21.98 \, m$

(c) $C = \pi d = 3.14 \ x \ 4.5 = 14.13 \, dm$

(d) $C = 2\pi r = 2 \ x \ 3.14 \ x \ 8 = 50.24 \, km$

Example 3

The circumference of a car wheel is 150 cm. What is the radius of the wheel?

Solution

Given circumference, C = 150 cm

$$C = 2\pi r$$

$$r = \frac{C}{2\pi} = \frac{150}{2 \, x \, 3.14} = \frac{150}{6.28} = 23.89 \, cm$$

∴ The radius of the wheel is 23.89 cm

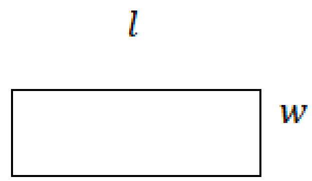
Areas of Rectangles and Triangles

The Area of a Rectangle

Calculate the area of a rectangle

Area – can be defined as the total surface covered by a shape. The shape can be rectangle, square, trapezium e. t. c. Area is measured in mm!, cm!,dm!,m! e. t. c

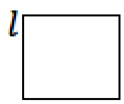
Consider a rectangle of length l and width w



$$\text{Area}, \mathrm{A} = l \ \mathrm{x} \ \mathrm{w} = l \, \mathrm{w}$$

$$\boxed{\mathrm{A} = lw}$$

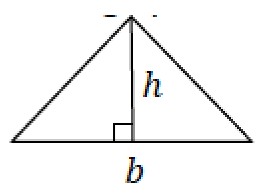
Consider a square of side l



$$\text{Area}, \mathrm{A} = l \, x \, l$$

$$\boxed{\mathrm{A} = l^2}$$

Consider a triangle with a height, h and a base, b



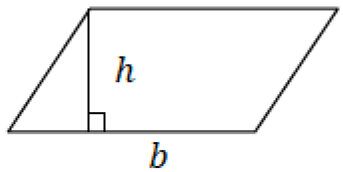
$$\boxed{Area, A = \frac{1}{2} hb}$$

Areas of Trapezium and Parallelogram

The Area of a Parallelogram

Calculate area of a parallelogram

A parallelogram consists of two triangles inside. Consider the figure below:

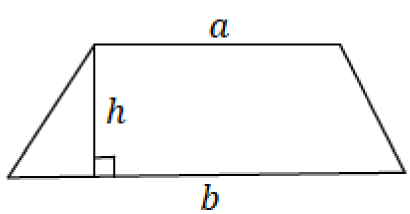


$$\boxed{Area, A = bh}$$

The Area of a Trapezium

Calculate the area of a trapezium

Consider a trapezium of height, h and parallel sides a and b



$$\boxed{Area, A = \frac{1}{2}h(a+b)}$$

Example 4

The area of a trapezium is120 m!. Its height is 10 m and one of the parallel sides is 4 m. What is the other parallel side?

Solution

Given area, A = 120 m^2, height, h = 10 m, one parallel side, a = 4 m. Let other parallel side be, b

Then

$$A = \frac{1}{2}h(a + b)$$

$$120 = \frac{1}{2}\ x\ 10\ x\ (4 + b)$$

$$120 = 5\ x\ (4 + b)$$

$$4 + b = \frac{120}{5}$$

$$4 + b = 24$$

$$b = 24 - 4 = 20$$

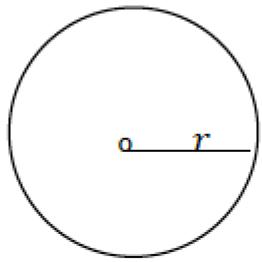
$\therefore$ The length of other parallel side is $20\ m$

Area of a Circle

Areas of Circle

Calculate areas of circle

Consider a circle of radius r;



$$\boxed{Area, A = \pi r^2}$$

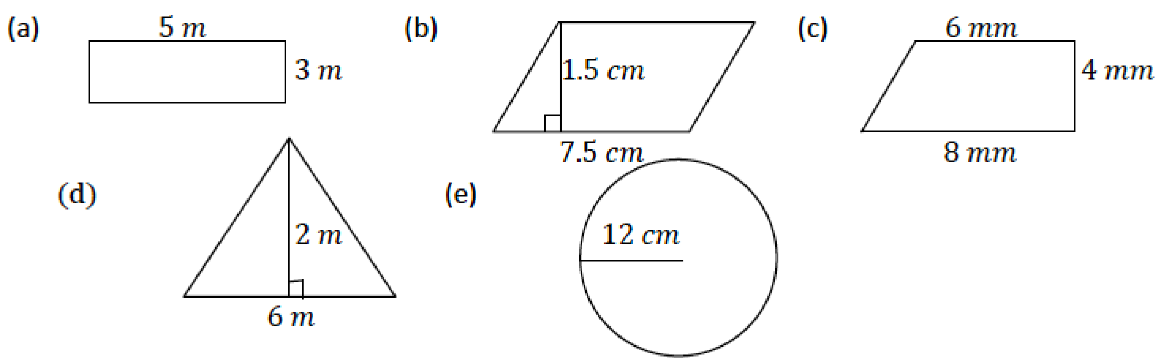
Also $r = \frac{d}{2}$, then

$$Area, A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$$\boxed{Area, A = \frac{\pi d^2}{4}}$$

Example 5

Find the areas of the following figures



Solution

(a) $A = lw = 5\ x\ \ 3 = 15\ m^2$

(b) $A = bh = 7.5\ x\ 1.5 = 11.25cm^2$

(c) $A = \frac{1}{2}h(a+b) = \frac{1}{2}\ x\ 4\ x\ (6+8) = \frac{1}{2}\ x\ 4\ x\ 14 = 28\ mm^2$

(d) $A = \frac{1}{2}hb = \frac{1}{2}\ \ x\ \ 2\ \ x\ \ 6 = 6\ m^2$

(e) $A = \pi r^2 = \frac{22}{7}\ \ x\ \ 12^2 = \frac{22}{7}\ \ x\ \ 144 = \frac{3168}{7} = 452.57\ cm^2$

Example 6

A circle has a circumference of 30 m. What is its area?

Solution

Given circumference, C = 30 m

C = $2\pi r$

$$r = \frac{C}{2\pi} = \frac{30}{2\ x\ 3.14} = \frac{30}{6.28} = 4.78\ m$$

Then

$$A = \pi r^2 = \frac{22}{7}\ \ x\ \ (4.78)^2 = \frac{22}{7}\ \ x\ \ 22.85 = \frac{502.7}{7} = 71.81\ m^2$$